Feedback Regulation of Blood Pressure in a Pulsatile Heart Model

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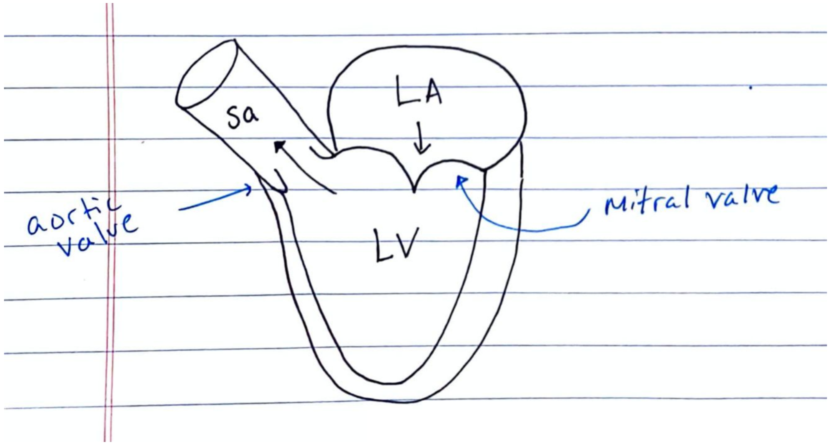
Final project for Special Topics in Modeling & Simulation Fall 2020 taught by Dr. Charles S. Peskin

Plan

- Introduce the pulsatile bloodflow model (uncontrolled BP)
- Look at an example of exercise to see the model’s limitation in this model
- Incorporate feedback aspect into the model
- Further improvements

Pulsatile Bloodflow Model: Uncontrolled Circulation

Scope of the model: **left ventricle** to **systemic arteries**



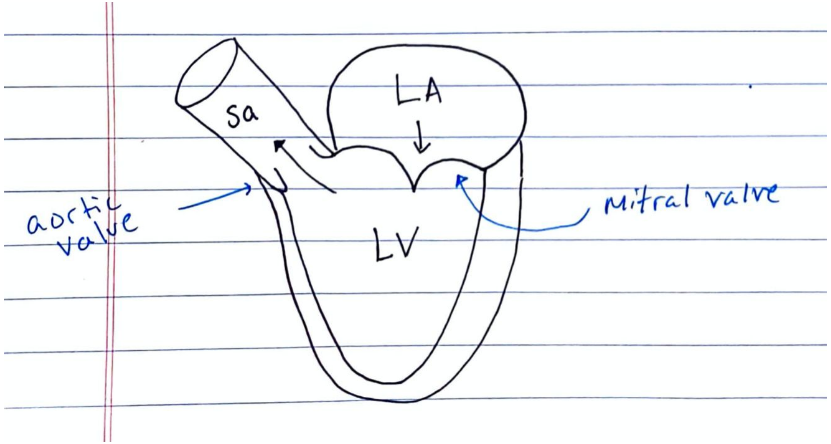
Some key definitions:

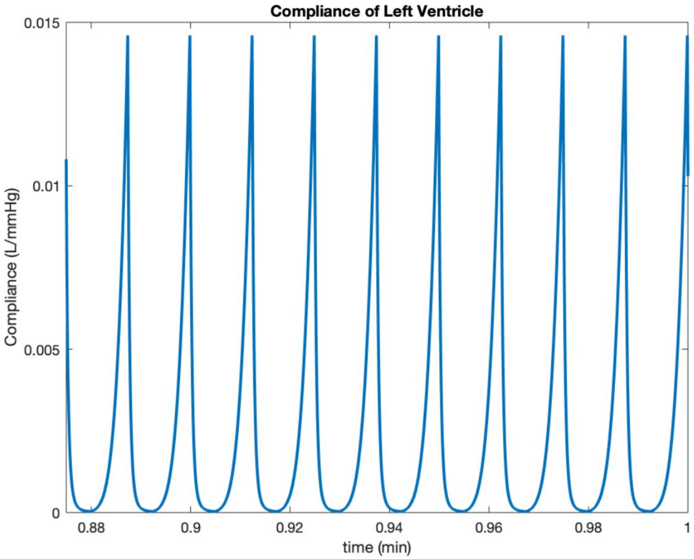
Systole = contraction
Diastole = relaxation
Heart rate = # of beats per minute (F)
Period = duration of heartbeat (T=1/F)
Cardiac output = volume of blood delivered to organs in one minute
Stroke volume = volume of blood ejected in one beat

$$\text{Cardiac Output} = \text{Stroke Volume} \times \text{Heart Rate}$$

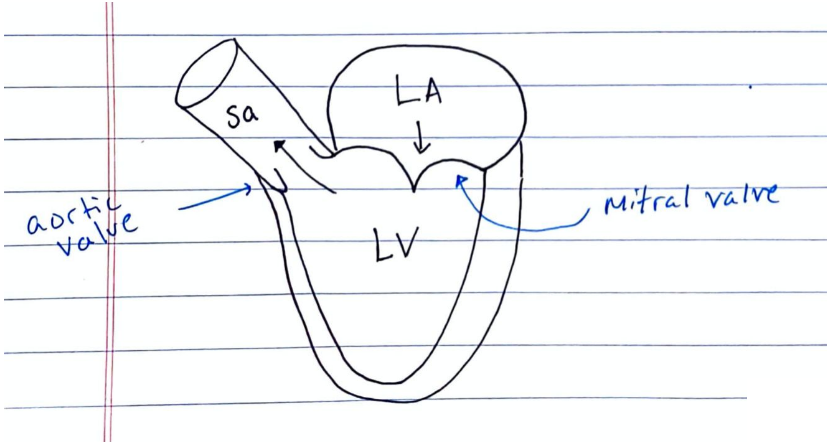
$$\frac{\text{volume}}{\text{minute}} = \frac{\text{volume}}{\text{beat}} \times \frac{\text{beats}}{\text{min}}$$

Derivation of the model: Left ventricle is a compliance chamber:





Derivation of the model: Blood flow equations



$$\frac{dV_{\mathrm{LV}}}{dt} = Q_{\mathrm{Mi}} - Q_{\mathrm{Ao}}$$

$$\frac{dV_{\mathrm{sa}}}{dt} = Q_{\mathrm{Ao}} - Q_{\mathrm{s}},$$

$$V_{\mathrm{LV}} = V_{\mathrm{d,LV}} + C_{\mathrm{LV}}(t) P_{\mathrm{LV}}$$

$$V_{\mathrm{sa}} = V_{\mathrm{d,sa}} + C_{\mathrm{sa}} P_{\mathrm{sa}}$$

$$Q_{\mathrm{Mi}} = \mathbf{1}_{\mathrm{Mi}} \frac{(P_{\mathrm{LA}} - P_{\mathrm{LV}})}{R_{\mathrm{Mi}}} \qquad \mathbf{1}_{\mathrm{Mi}} = \begin{cases} 0 & P_{\mathrm{LA}} < P_{\mathrm{LV}} \\ 1 & P_{\mathrm{LA}} > P_{\mathrm{LA}} \end{cases}$$

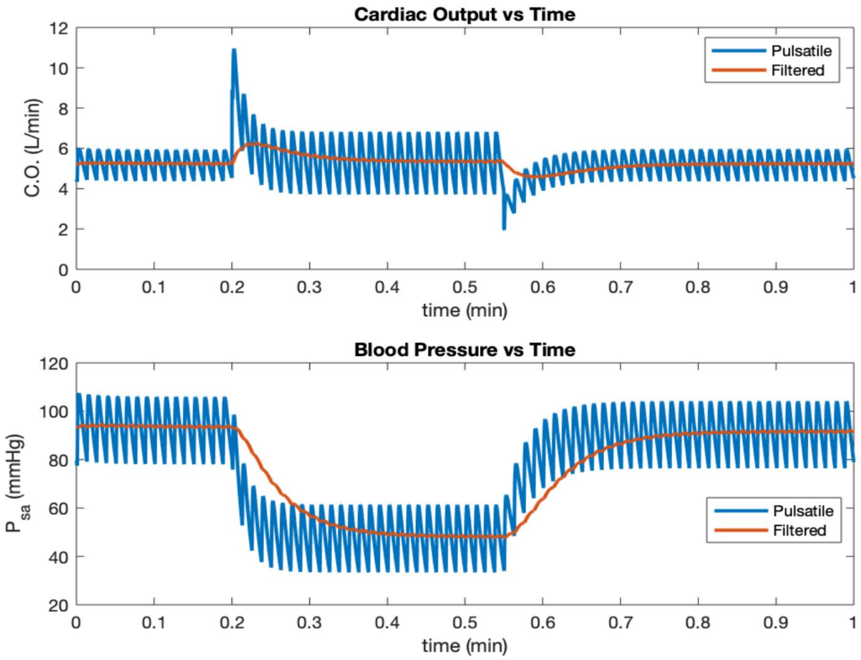
$$Q_{\mathrm{Ao}} = \mathbf{1}_{\mathrm{Ao}} \frac{(P_{\mathrm{LV}} - P_{\mathrm{sa}})}{R_{\mathrm{Ao}}} \qquad \mathbf{1}_{\mathrm{Ao}} = \begin{cases} 0 & P_{\mathrm{LV}} < P_{\mathrm{sa}} \\ 1 & P_{\mathrm{LV}} > P_{\mathrm{sa}} \end{cases}$$

$$Q_{\mathrm{s}} = \frac{P_{\mathrm{sa}}}{R_{\mathrm{s}}}$$

What happens during exercise?

Blood vessels dilate, Heart rate and cardiac output rise ---> Blood Pressure rises

- We introduce exercise in our model by decreasing the systemic resistance:



$$\tau_P \frac{dP_{\text{filtered}}}{dt} = P_{\text{sa}} - P_{\text{filtered}}$$

$$\tau_Q \frac{dQ_{\text{filtered}}}{dt} = Q_{\text{s}} - Q_{\text{filtered}}$$

Fix: Model the effects of baroreceptor loop

-During exercise, the brain controls our blood pressure by increasing the heart rate.

-Model detects drop in BP and makes adjustments to heart rate, F (previously parameter):

$$F(t) = F_{\text{heart}} + \Delta F \qquad F_{\text{heart}} = 1/T_{\text{heart}}$$

$$\Delta F = cF_{\text{heart}} \frac{P_{\text{filtered}} - P^*}{P^*} \quad \text{where } c < 0$$

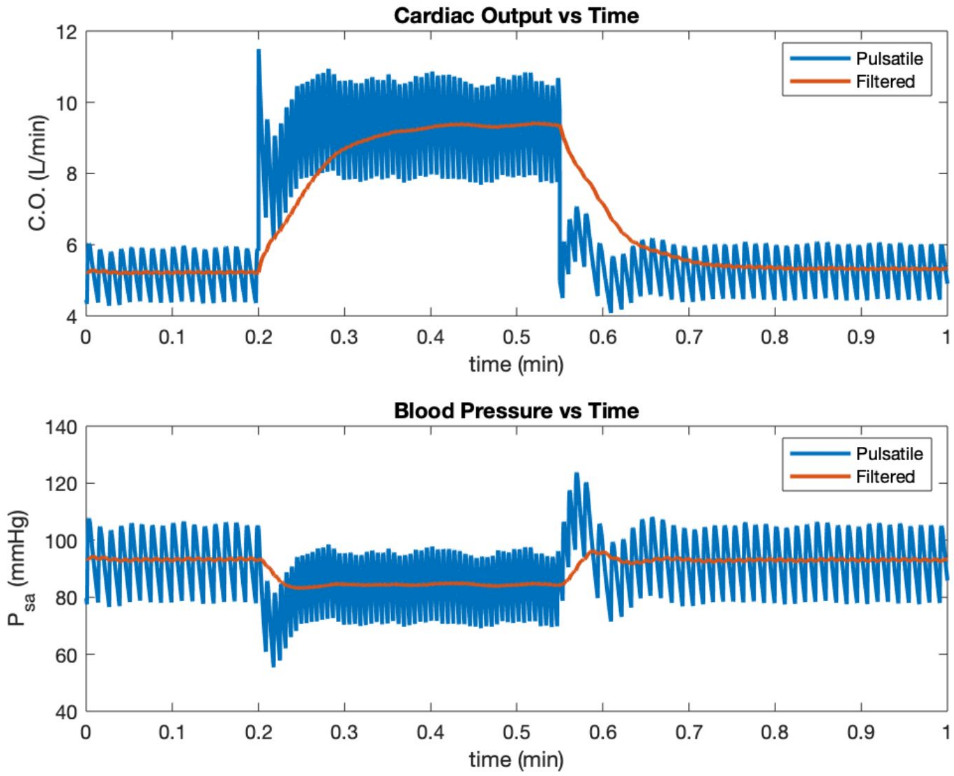
$P_{\text{filtered}} < P^*$, we will have an increase in heartrate.

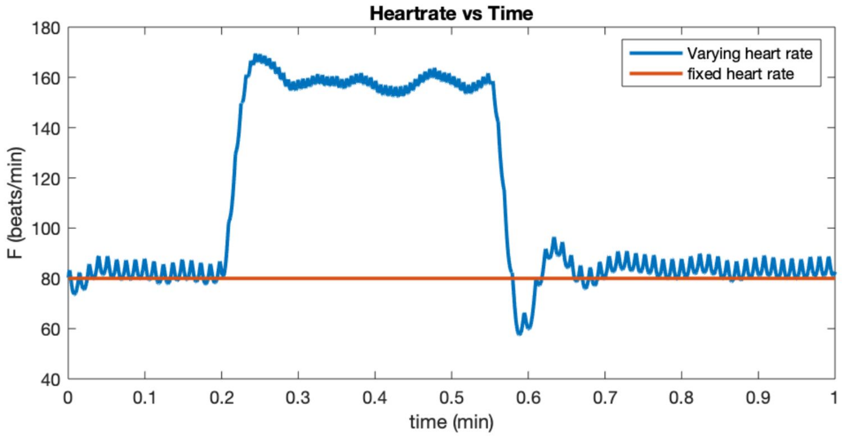
The compliance function has to detect this changing heart rate:

$$C(t) = C_{\text{LV}}(t_{\text{heart}}(t))$$

$$\frac{dt_{\text{heart}}}{dt} = \frac{F(t)}{F_{\text{heart}}}$$

Controlled Circulation Results (almost there!):





To fix the blood pressure, we need to change the set pressure P^* as we alter systemic resistance to start/stop exercise: