Feedback Control Model of the Heart During Exercise

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Abstract

This report describes an improvement to a compartmental pulsatile heart model. The original model without any modifications is not equipped with the appropriate apparatus to accurately simulate physiological changes as a result of dynamical perturbations such as exercise. A feedback controller that regulates blood pressure was implemented to model the appropriate physiological changes. The fundamental change made to the original model is that the heart rate will be a discrete time-variable rather than a constant parameter. A differential equation model for varying heart rate using a time-axis warping compliance function was formulated and used to understand paradoxical relationship between blood pressure and systemic resistance.

1 Introduction

During exercise, various changes to cardiovascular function occur. Blood pressure and heart rate rise and the blood vessels dilate to allow for the blood to flow more freely and quickly to the active muscles so that oxygen can be delivered there. In the pulsatile blood flow model for the heart, there is no feedback control mechanism to simulate something like exercise. The goal of this project is to introduce the uncontrolled model and its limitations and then modify it to incorporate feedback control of blood pressure and compare the results to physiological expectations.

2 Mathematical Model for Pulsatile Blood Flow

The heart pumps blood into the arteries in discrete surges during contractions, causing the blood pressure to rise and fall periodically. Therefore, in this model, the pressures, flows, and volumes are taken to be time-dependent functions $P(t), Q(t)$, and $C(t)$ respectively. The flow is the volume of blood passing through a certain point per unit time. Compliance is the elastic deformation property of a chamber. When the ventricles pump, they go through two phases during one cardiac cycle: systole, where the ventricular muscles contract to pump blood into the lungs and organs, and diastole, where the ventricular muscles relax and allow for blood to fill up the chamber again. Consider a compliance vessel with values for inflow $Q_1(t)$ and outflow $Q_2(t)$. If the vessel is not in a steady state, then these two values are unequal and the volume of the vessel is no longer a constant. Let the volume of a vessel be

regarded as $V(t)$ at a time t. We say the rate of change of volume with respect to time is the difference between the inflow and outflow of the vessel:

$$\frac{dV}{dt} = Q_1 - Q_2. \tag{1}$$

The compliance equation can be used to relate volume as so:

$$V(t) = CP(t) + V_{\mathrm{d}}. \tag{2}$$

where V_{d} is known as the dead volume which is the residual volume when the pressure is zero. Deriving this equation with respect to time yields the equality:

$$C\frac{dP}{dt} = Q_1 - Q_2, \tag{3}$$

a convenient differential equation that accounts for pressure as a function of time in a compliance chamber with unsteady flow.

2.1 Uncontrolled Circulation

The scope of the model will focus on just the left ventricle and the systemic arteries. We consider both of these to be compliance chambers connected via a resistance element. The left ventricle is equipped with an inflow valve called the mitral valve and an outflow valve called the aortic valve. A healthy valve permits flow in a single direction with low resistance and blocks flow in the reverse direction. This is illustrated in figure 1.

To incorporate these components into our model, we begin by introducing the equations for volume conservation in each chamber:

$$\frac{dV_{\mathrm{LV}}}{dt} = Q_{\mathrm{Mi}} - Q_{\mathrm{Ao}} \tag{4}$$

$$\frac{dV_{\mathrm{sa}}}{dt} = Q_{\mathrm{Ao}} - Q_{\mathrm{s}}, \tag{5}$$

as well as the equations for the compliance relations for each chamber:

$$V_{\mathrm{LV}} = V_{\mathrm{d,LV}} + C_{\mathrm{LV}}(t)P_{\mathrm{LV}} \tag{6}$$

$$V_{\mathrm{sa}} = V_{\mathrm{d,sa}} + C_{\mathrm{sa}}P_{\mathrm{sa}} \tag{7}$$

Recall from earlier that the compliance is related to the elasticity of a vessel. Since the arteries and veins do not exhibit much change in this property during a cardiac cycle, C_{sa} is taken to be constant and here we are considering C_{LV} to be a varying function of time since the ventricle goes through periodic contracting and relaxing phases. Next, we define flow equations that take into consideration the pressure differences to determine whether the valve in question is open or closed. For each of the valve flows we define indicator variables $\mathbf{1}_{\mathrm{Mi}}$ and $\mathbf{1}_{\mathrm{Ao}}$ which take on either values of 1 or 0 to indicate that the corresponding valve is

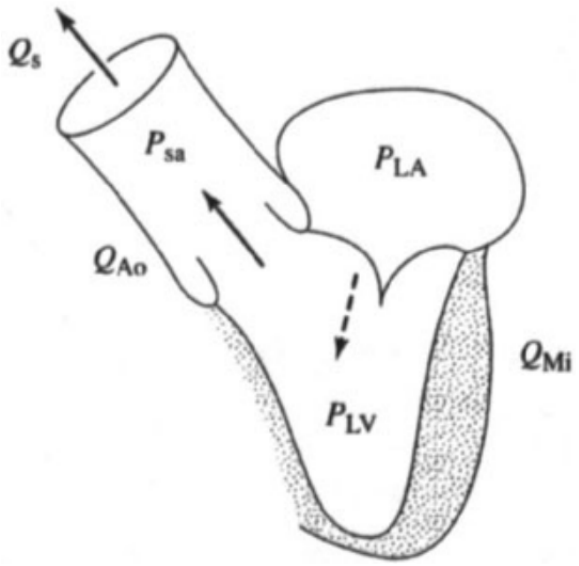

Figure 1: Diagram of the left heart and systemic artery connectivity and bloodflow directionality. Pressures and flows are P_{LA} = left atrial pressure, P_{LV} = left ventricular pressure, P_{sa} = systemic arterical pressure, Q_{Mi}; = flow through the mitral valve (indicated here by a dotted arrow because the mitral valve is closed at the moment shown), Q_{Ao} = flow through the aortic valve, and Q_{s} = outflow from the systemic arterial tree, i.e., aggregate flow through all of the tissues of the body.

open or closed respectively.

$$Q_{\mathrm{Mi}} = \mathbf{1}_{\mathrm{Mi}} \frac{(P_{\mathrm{LA}} - P_{\mathrm{LV}})}{R_{\mathrm{Mi}}} \tag{8}$$

$$Q_{\mathrm{Ao}} = \mathbf{1}_{\mathrm{Ao}} \frac{(P_{\mathrm{LV}} - P_{\mathrm{sa}})}{R_{\mathrm{Ao}}} \tag{9}$$

$$Q_{\mathrm{s}} = \frac{P_{\mathrm{sa}}}{R_{\mathrm{s}}} \tag{10}$$

where R_s is the systemic resistance, R_{Ao} is the resistance of the aortic valve, and R_{Mi} is the resistance of the mitral valve. The valve states are determined by the pressure differences between the vessels that the valve connects. If the pressure difference is positive (in the direction that blood is supposed to flow) then the valve is open and if it is negative then the valve is closed:

$$\mathbf{1}_{\mathrm{Mi}} = \begin{cases} 0 & P_{\mathrm{LA}} < P_{\mathrm{LV}} \\ 1 & P_{\mathrm{LA}} > P_{\mathrm{LA}} \end{cases} \tag{11}$$

$$\mathbf{1}_{\mathrm{Ao}} = \begin{cases} 0 & P_{\mathrm{LV}} < P_{\mathrm{sa}} \\ 1 & P_{\mathrm{LV}} > P_{\mathrm{sa}} \end{cases} \tag{12}$$

In the special case that the upstream and downstream pressures of the valve are equal, it makes no difference whether we regard the valve as closed or open since either way the flow will be zero.

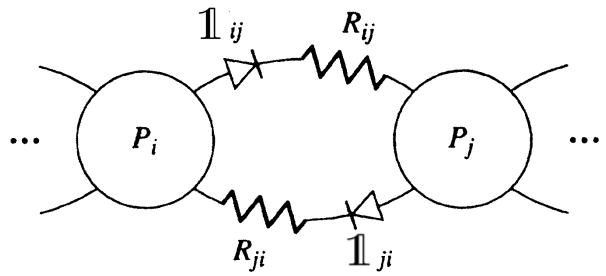

Figure 2: Part of a general model of the whole circulation. A pair of compliance chambers is shown, each chamber labeled by its pressure P_i or P_j. Every such pair in the model is connected by a pair of resistance vessels equipped with valves, the two valves of the pair pointing in opposite directions. $S_{ji} = 0$ or 1, denoting the state of the valve that allows flow from compliance chamber i into compliance chamber j. R;i = resistance of the vessel in series with this valve.

Now the model for the left heart and the systemic arteries consists of nine equations. The nine unknown functions of time are the left ventricular and systemic arterial volumes and pressures V_{LV}, V_{sa} P_{LV}, P_{sa}, the flows through the mitral valve, aortic valve, systemic resistance Q_{Mi}, Q_{Ao}, Q_{s}, and the indicator variables for the mitral and aortic valves $\mathbf{1}_{\mathrm{Mi}}$ and $\mathbf{1}_{\mathrm{Ao}}$. We take the left ventricular compliance C_{LV} to be a given function of time. The constant parameters of this set up are left atrial pressure P_{LA}, mitral and aortic valve resistances R_{Mi} and R_{Ao}, systemic resistance R_{s}, and systemic arterial compliance C_{sa}. The left ventricular compliance is periodic with period T and expressed in a way that considers time zero to be at end-diastole and thus at a maximum:

$$C_{\mathrm{LV}}(t+T) = C_{\mathrm{LV}}(t) \tag{13}$$

$$C_{\mathrm{LV}}(t) = \begin{cases} C_{\mathrm{LVD}}\left(\frac{C_{\mathrm{RVS}}}{C_{\mathrm{LVD}}}\right)^{\frac{1-exp(-t/\tau_{\mathrm{S}})}{1-exp(-T_{\mathrm{S}}/\tau_{\mathrm{S}})}} & 0 \le t \le T_{\mathrm{S}} \\ C_{\mathrm{LVS}}\left(\frac{C_{\mathrm{LVD}}}{C_{\mathrm{LVS}}}\right)^{\frac{1-exp(-(t-T_{\mathrm{S}})/\tau \mathrm{D})}{1-exp(-(T-T_{\mathrm{S}})/\tau_{\mathrm{S}})/\tau_{\mathrm{D}}}} & T_{\mathrm{S}} \le t \le T \end{cases} \tag{14}$$

This form of the compliance equation for the left ventricle shows us the transition from maximum value C_{LVD} to minimum value C_{LVS} and the speed of these transitions is inversely proportional to the time constants τ_S and τ_D.

2.2 Numerical Methods for Uncontrolled Pulsatile Model

To solve the nine equations for the left heart and systemic arteries, we simplify them to express everything in terms of pressure.

$$\frac{d(C_{\mathrm{LV}}P_{\mathrm{LV}})}{dt} = \frac{\mathbf{1}_{\mathrm{Mi}}(P_{\mathrm{LA}} - P_{\mathrm{LV}})}{R_{\mathrm{Mi}}} - \frac{\mathbf{1}_{\mathrm{Ao}}(P_{\mathrm{LV}} - P_{\mathrm{sa}})}{R_{\mathrm{Ao}}} \tag{15}$$

$$C_{\mathrm{sa}}\frac{dP_{\mathrm{sa}}}{dt} = \frac{\mathbf{1}_{\mathrm{Ao}}(P_{\mathrm{LV}} - P_{\mathrm{sa}})}{R_{\mathrm{Ao}}} - \frac{P_{\mathrm{sa}}}{R_{\mathrm{s}}} \tag{16}$$

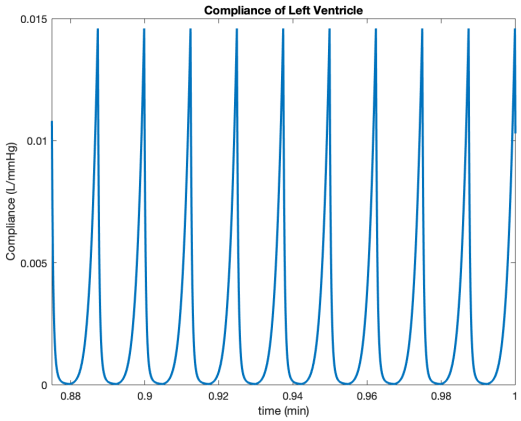

Figure 3: Compliance vs. Time over the last ten cardiac cycles of an arbitrary simulation. We can see the oscillations between maximum compliance at diastole and minimum compliance at systole.

The systemic arterial compliance was factored out of the time derivative because we noted earlier that it is a constant. Next, the backwards Euler method can be used as it was before to numerically solve the differential equations:

$$\frac{C_{\mathrm{LV}}(t)P_{\mathrm{LV}}(t) - C_{\mathrm{LV}}(t-\Delta t)P_{\mathrm{LV}}(t-\Delta t)}{\Delta t} \tag{17}$$

$$= \frac{\mathbf{1}_{\mathrm{Mi}}(t)}{R_{\mathrm{Mi}}}(P_{\mathrm{LA}} - P_{\mathrm{LV}}(t)) - \frac{\mathbf{1}_{\mathrm{Ao}}(t)}{R_{\mathrm{Ao}}}(P_{\mathrm{LV}}(t) - P_{\mathrm{sa}}(t)), \tag{18}$$

$$C_{\mathrm{sa}}\frac{P_{\mathrm{sa}}(t) - P_{\mathrm{sa}}(t-\Delta t)}{\Delta t} \tag{19}$$

$$= \frac{\mathbf{1}_{\mathrm{Ao}}(t)}{R_{\mathrm{Ao}}}(P_{\mathrm{LA}} - P_{\mathrm{sa}}(t)) - \frac{P_{\mathrm{sa}}}{R_{\mathrm{s}}}(t). \tag{20}$$

Consider this to be a 2×2 linear system where the tricuspid and aortic valve states are known. We use this assumption to solve for the pressures of the left ventricle and systemic arteries in terms of the tricuspid and aortic valve indicator functions. We begin by rewriting the above equations into standard form:

$$C_{11}(t)P_{\mathrm{LV}}(t) + C_{12}(t)P_{\mathrm{sa}}(t) = B_1(t), \tag{21}$$

$$C_{21}(t)P_{\mathrm{LV}}(t) + C_{22}(t)P_{\mathrm{sa}}(t) = B_2(t) \tag{22}$$

where we let:

$$C_{11}(t) = C_{\mathrm{LV}}(t) + \Delta t(\frac{\mathbf{1}_{\mathrm{Mi}}(t)}{R_{\mathrm{Mi}}} + \frac{\mathbf{1}_{\mathrm{Ao}}(t)}{R_{\mathrm{Ao}}}) \tag{23}$$

$$C_{12}(t) = C_{21}(t) = -\Delta t(\frac{\mathbf{1}_{\mathrm{Ao}}(t)}{R_{\mathrm{Ao}}}) \tag{24}$$

$$C_{22}(t) = C_{\mathrm{sa}} + \Delta t(\frac{\mathbf{1}_{\mathrm{Ao}}(t)}{R_{\mathrm{Ao}}} + \frac{1}{R_{\mathrm{s}}}), \tag{25}$$

$$B_1(t) = C_{\mathrm{LV}}(t - \Delta t)P_{\mathrm{LV}}(t - \Delta t) + \Delta t\frac{\mathbf{1}_{\mathrm{Mi}}(t)}{R_{\mathrm{Mi}}}P_{\mathrm{LA}}, \tag{26}$$

$$B_2(t) = C_{\mathrm{sa}}P_{\mathrm{sa}}(t - \Delta t) \tag{27}$$

The solution in terms of the determinant $D(t)$ of the 2×2 matrix is as follows:

$$P_{\mathrm{LV}}(t) = \frac{B_1(t)C_{22}(t) - B_2(t)C_{12}(t)}{D(t)} \tag{28}$$

$$P_{\mathrm{sa}}(t) = \frac{B_2(t)C_{22}(t) - B_1(t)C_{21}(t)}{D(t)} \tag{29}$$

$$D(t) = C_{11}(t)C_{22}(t) + C_{12}(t)C_{21}(t) \tag{30}$$

It is necessary to show that the determinant is strictly greater than zero in order to confirm there is never a zero in the denominator. It is sufficient to show that

$$C_{11}(t)C_{22}(t) > C_{12}(t)C_{21}(t) \tag{31}$$

$$\tag{32}$$

Rewriting the left hand side yields:

$$[C_{\mathrm{LV}}(t) + \Delta t\frac{\mathbf{1}_{\mathrm{Mi}}(t)}{R_{\mathrm{Mi}}} + \Delta t\frac{\mathbf{1}_{\mathrm{Ao}}(t)}{R_{\mathrm{Ao}}}] * [C_{\mathrm{sa}} + \Delta t\frac{\mathbf{1}_{\mathrm{Ao}}(t)}{R_{\mathrm{Ao}}} + \Delta t\frac{1}{R_{\mathrm{s}}}] \tag{33}$$

$$= (\Delta t\frac{\mathbf{1}_{\mathrm{Ao}}(t)}{R_{\mathrm{Ao}}})^2 + \sigma > (\Delta t\frac{\mathbf{1}_{\mathrm{Ao}}(t)}{R_{\mathrm{Ao}}})^2 = C_{12}C_{21} \tag{34}$$

where σ is a positive value resulting from distributing the product on the left hand side.

3 Results for Uncontrolled Circulation during Exercise

Cardiac output is defined as the volume of blood delivered to the organs per minute and is given by the following:

$$Q_s = V_s \times F = V_s/T \tag{35}$$

where V_s is the stroke volume (blood volume delivered to organs per beat) and F is the heart rate in beats per minute and the reciprocal of the period T. During exercise, the arterioles in the exercising muscle dilate, which results in a drop in systemic resistance R_s. The cardiac output (Q_s) rises primarily due to an increase in heart rate as does the blood pressure

P_{sa}. This section will study the effects of the uncontrolled pulsatile model in response to a reduction in R_s.

For one, we know that the heart rate in our model is constant, and thus we should not expect the cardiac output to change much. Also, by Ohm's law, if the systemic resistance goes down, so should the systemic arterial pressure. In the pulsatile model, let $t_{\text{start}} = 0.20$ be the time that we begin exercise and let $t_{\text{end}} = 0.55$ be the time that we stop exercise. The starting and stopping of exercise are introduced as a step change (decrease and increase, respectively) in R_s at those times by a factor of roughly 2. In order to analyze trends in the pulsatile data, which is quite oscillatory, we can apply a low-pass filter on cardiac output and blood pressure to get a more steady signal, which will prove to be quite useful later on especially for blood pressure:

$$\tau_P \frac{dP_{\text{filtered}}}{dt} = P_{\text{sa}} - P_{\text{filtered}} \tag{36}$$

$$\tau_Q \frac{dQ_{\text{filtered}}}{dt} = Q_{\text{s}} - Q_{\text{filtered}} \tag{37}$$

where τ_P and τ_Q are time constants for the respective filters. We can simply use forward Euler to get an explicit expression for the filtered values at each time step:

$$\tau_P \frac{P_{\text{filtered}}(t + \Delta t) - P_{\text{filtered}}(t)}{\Delta t} = P_{\text{sa}}(t) - P_{\text{filtered}}(t) \tag{38}$$

$$\tau_Q \frac{Q_{\text{filtered}}(t + \Delta t) - Q_{\text{filtered}}(t)}{\Delta t} = Q_{\text{sa}}(t) - Q_{\text{filtered}}(t) \tag{39}$$

In figure 4 we can see that (by following the red line for the filtered values) as exercise begins with the step reduction of R_{s} there is also a sharp drop in blood pressure and a slight increase in cardiac output that dies back down before exercise nears its end. This is quite far from what really happens in exercise and this is a problem we will attempt to solve in the subsequent sections.

4 Feedback System for Controlled Circulation

We look to systematically fix the problems in our model that were made apparent in the previous subsection. Our goal is to see a larger increase in cardiac output than what we saw before and also to have an increase in blood pressure.

4.1 Varying Heartrate Function

The main issue in the uncontrolled circulation model was that we fixed the heartrate F to be some constant value through out the duration of the simulation. This resulted in discrepancies between the real physiology and the model. The solution is to implement a time-varying function $F(t)$ for continuously varying heartrate. The feedback system that controls heartrate is a baroreceptor (pressure detector) loop based on some established neural control. Thus, we want our model to detect when exercise begins and adjust heartrate

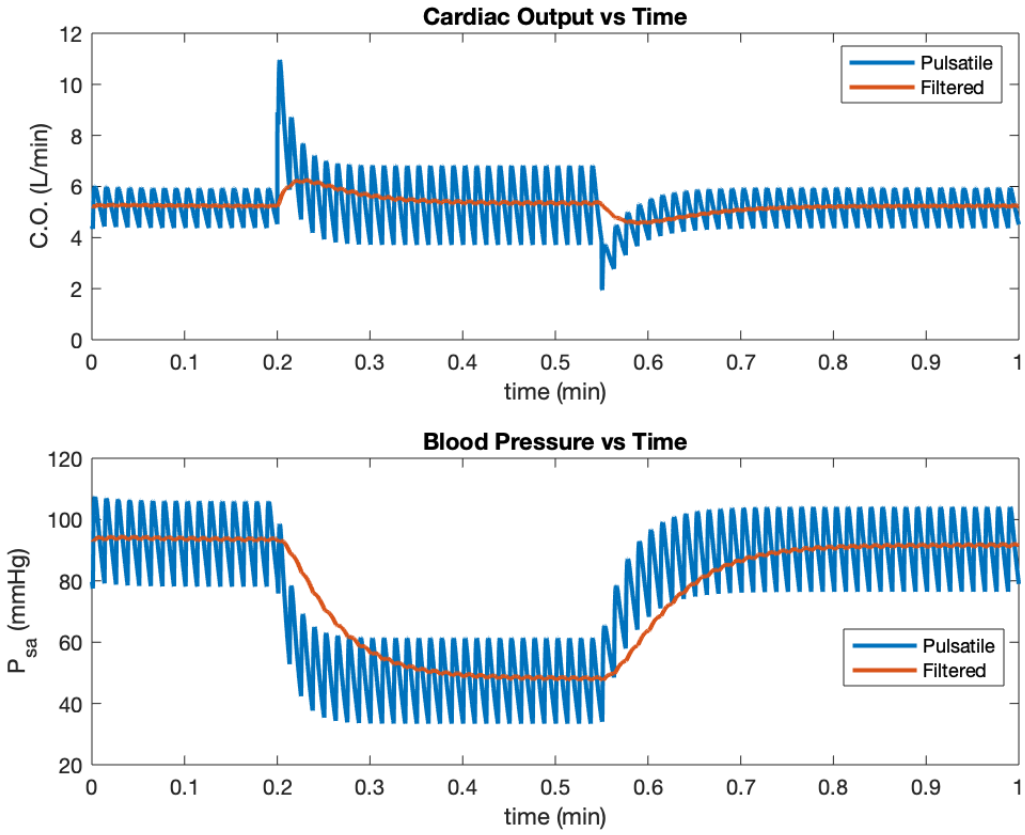

Figure 4: Cardiac output (systemic flow, Q_s) plotted as a function of time and systemic arterial pressure plotted as a function of time. Here we can see that when exercise was introduced at $t_{\text{start}} = 0.2$, blood pressure dropped and cardiac output only modestly increased.

accordingly. When we drop our systemic resistance, the first thing that happens is the blood pressure lowers due to Ohm's Law (10). Let P^* be some set pressure, perhaps the systemic arterial mean pressure for normal parameters, and compare the filtered pressure P_{filtered} to it at every time step and inversely change the heartrate based on this:

$$F(t) = F_{\text{heart}} + \Delta F \tag{40}$$

$$\Delta F = cF_{\text{heart}} \frac{P_{\text{filtered}} - P^*}{P^*} \tag{41}$$

where $c < 0$ is acting as the gain of this feedback system and F_{heart} is the initial steady heartrate. Note that whenever $P_{\text{filtered}} < P^*$, we will have an increase in heartrate.

The next step is to implement $F(t)$ into our compliance function which drives our model. To do this, we consider our known periodic function for compliance, $C_{\text{heart}}(t_{\text{heart}})$ with period T_{heart} and heart rate $F_{\text{heart}} = 1/T_{\text{heart}}$. To produce continuously varying heartrate $F(t)$, we set our new compliance function to look like:

$$C(t) = C_{\text{LV}}(t_{\text{heart}}(t)) \tag{42}$$

where we can solve for t_{heart} with the following ODE:

$$\frac{dt_{\text{heart}}}{dt} = \frac{F(t)}{F_{\text{heart}}} \tag{43}$$

4.2 Results for a Controlled Circulation during Exercise

Here, we run the same experiment as we did in the uncontrolled circulation model. Now we should see an increase in heartrate when we decrease R_{s} as a response to the blood pressure drop and as a result. This will in turn cause the cardiac output to rise.

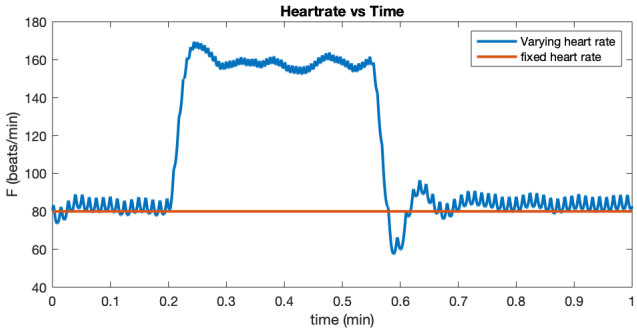

Figure 5: Continuous heartrate as a function of time compared to the set steady heartrate. R_{s} is decreased by a factor of 2 at $t_{\text{start}} = 0.2$ and restored at $t_{\text{end}} = 0.55$

There is still, however, a limitation with this controlled model. Note in figure 6 that while cardiac output is increasing by nearly 100%, blood pressure is decreasing still, just not as much as it was in the uncontrolled model. This is a still a big improvement, but we can do better!

4.3 Resolution of a paradox concerning the sign of the error signal for feedback during exercise.

The reason that the blood pressure sees a decrease during exercise in this partially controlled model is that when systemic resistance drops, our set pressure P^* stays the same. This is not the same as what happens in the physiology. When we plan to initiate exercise, our brain essentially knows this and prepares for it and we model this by raising P^* simultaneously as we drop R_{s} at t_{start}. The important thing to note here is that when we stop exercise in the model at t_{end} we first need to change the set pressure back to the steady mean systemic arterial pressure. We cannot however restore our systemic resistance at this time, since this will cause a huge jump in blood pressure at the stop of exercise (again due to Ohm's Law). Instead we restore the systemic resistance at $t_{\text{end}} + \tau_e nd$ where $\tau_e nd$ is a small time constant. It turns out that a good choice for this time constant is the time constant we used for the low-pass-filter on blood pressure, τ_P.

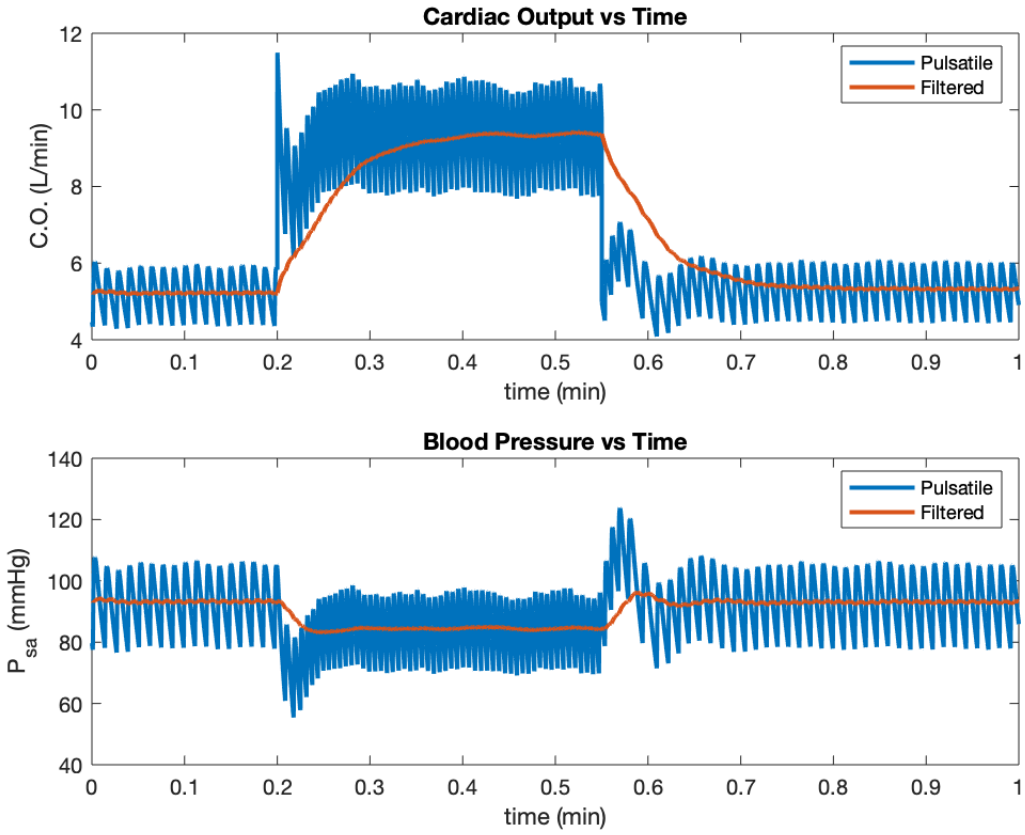

Figure 6: Effects on time course for cardiac output and blood pressure during exercise. Reduction in systemic resistance occurs at $t_{\text{start}} = 0.2$ and is restored at $t_{\text{end}} = 0.55$. Cardiac output nearly doubles during exercise while blood pressure slightly falls.

Now we should expect the model to resemble the physiology rather closely. The heartrate (and thus cardiac output) as well as the blood pressure should both rise significantly when exercise begins and return to normal when exercise ends. See figure 7

The compliance function that detects this changing heart rate looks like a distorted version of the original compliance function that detects a constant heartrate. This distortion is in the time axis. See figure 8 and 9 for the partially controlled model with constant set pressure and the fully controlled model with varying set pressure.

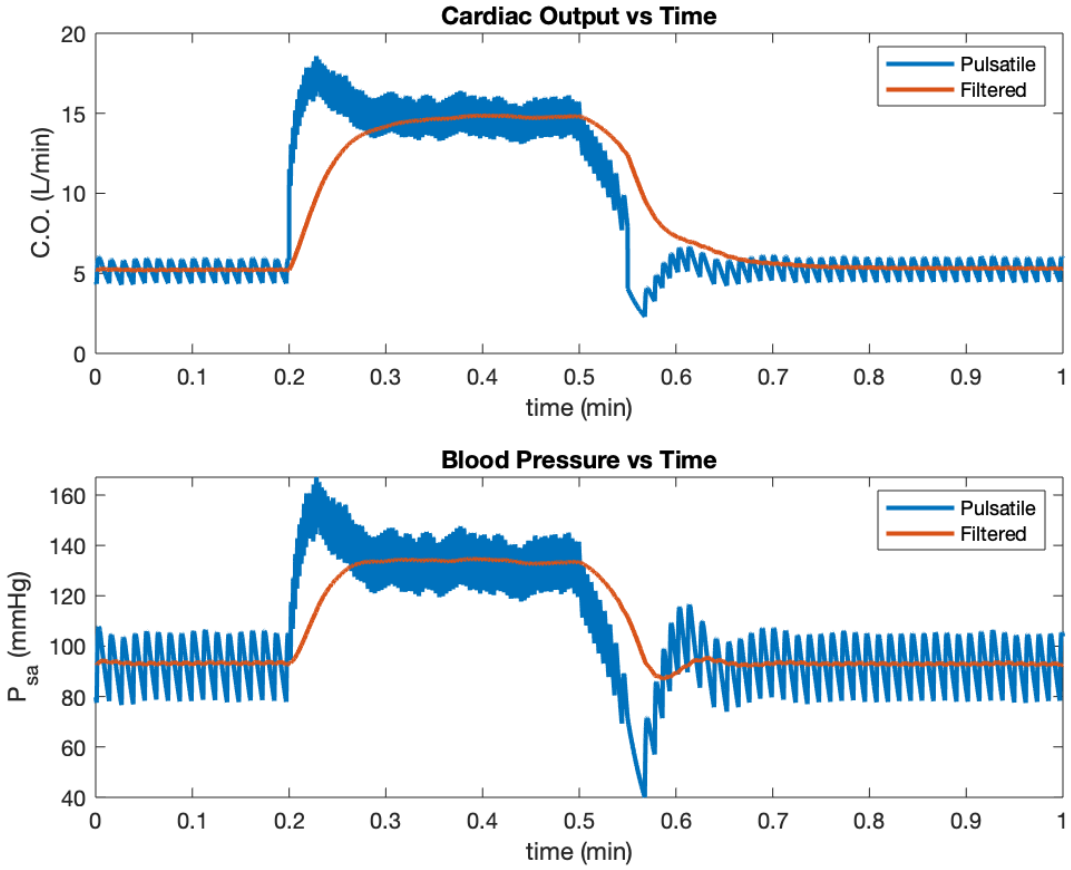

Figure 7: Effects on time course for cardiac output and blood pressure during exercise. Reduction by factor 2 in systemic resistance occurs at $t_{\text{start}} = 0.2$ along with an increase in set pressure P^* by roughly a factor of 2. $P^*$is restored at $t_{\text{end}} = 0.55$ and R_{s} is restored at $t_{\text{end}} + \tau_P$. Cardiac output roughly triples during exercise and blood pressure increases significantly as well.

5 Implementation in MATLAB

There are various programs involved in this simulation. We have a function for the compliance, a function to compute the pressures and valve states, and a driver script from which we run the simulation that we can toggle between controlled and uncontrolled models with. The following subsections outline in detail how these programs work.

5.1 CV_now.m

This is a function that implements the known compliance function we saw eariler. We use this compliance value to solve for all the other unknowns such as pressure, volume, and flow.

```
function CV=CV_now(t,CVS,CVD)
```

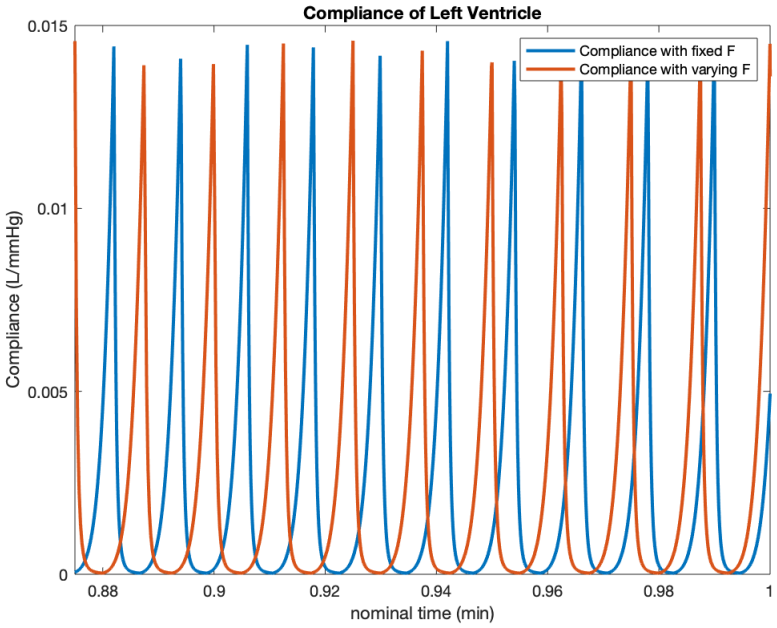

Figure 8: Compliance as a function of time for the last 10 cardiac cycles of the controlled circulation with constant P^*. We see shifts in the peaks indicating that the diastolic and systolic phases are occurring at different times for the two compliance functions.

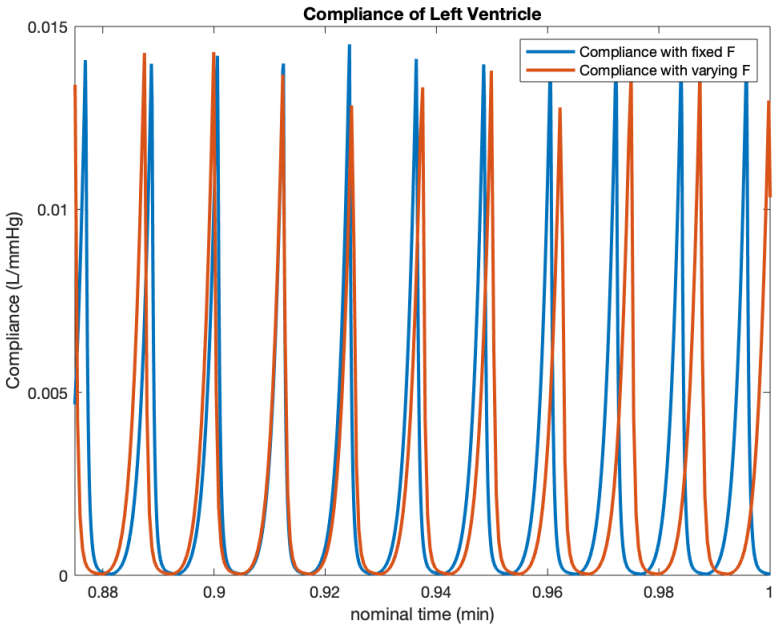

Figure 9: Compliance as a function of time for the last 10 cardiac cycles. We see shifts in the peaks indicating that the diastolic and systolic phases are occurring at different times for the two compliance functions.

```
%filename: CV_now.m
global T TS tauS tauD;
tc=rem(t,T); %tc=time in the current cycle,
             %measured from start of systole.
if(tc<TS)
  e=(1-exp(-tc/tauS))/(1-exp(-TS/tauS));
  CV=CVD*(CVS/CVD)^e;
else
  e=(1-exp(-(tc-TS)/tauD))/(1-exp(-(T-TS)/tauD));
  CV=CVS*(CVD/CVS)^e;
end
```

5.2 PLV_Psa_new.m

This is a function that solves for the pressures under the assumption that the valve states are known:

```
function [PLV,Psa]=PLV_Psa_new(PLV_old,Psa_old,CLV_old,CLV,SMi,SAo)
%filename PLV_Psa_new.m
global Csa Rs RMi RAo dt CHECK PLA;
C11=CLV+dt*((SMi/RMi)+(SAo/RAo));
C12=-dt*(SAo/RAo);
C22=Csa+dt*((SAo/RAo)+(1/Rs));
B1=CLV_old*PLV_old+dt*(SMi/RMi)*PLA;
B2=Csa*Psa_old;
D=C11*C22-C12^2;
PLV=(B1*C22-B2*C12)/D;
Psa=(B2*C11-B1*C12)/D;
if (CHECK)
  LHS1=(CLV*PLV-CLV_old*PLV_old)/dt;
  RHS1=(SMi/RMi)*(PLA-PLV)-(SAo/RAo)*(PLV-Psa);
  CH1=RHS1-LHS1;
  LHS2=Csa*(Psa-Psa_old)/dt;
  RHS2=(SAo/RAo)*(PLV-Psa)-(1/Rs)*Psa;
  CH2=RHS2-LHS2;
end
```

5.3 set_SMi_SAo.m

This script is setting the valve states, which uses the pressures. Since the pressures rely on the valve states, we use trial and error to find self consistent valve states and pressures.

```
%filename: set_SMi_SAo.m
%script to find
%self-consistent valve states and pressures:
done=0; %not done yet!
```

```
while(~done) %keep trying if not done (see below)
  SMi_noted=SMi; %note the value of SMi
  SAo_noted=SAo; %note the value of SAo
  % set pressures based on valve states:
  [PLV,Psa]=PLV_Psa_new(PLV_old,Psa_old,CLV_old,CLV,SMi,SAo);
  %and then set valve states based on pressures:
  SMi=(PLA>PLV); %evaluates to 1 if PLA>PLV, 0 otherwise
  SAo=(PLV>Psa); %evaluates to 1 if PLV>Psa, 0 otherwise
  %we're done if both valve states are unchanged:
  done=(SMi==SMi_noted)&(SAo==SAo_noted);
end
```

5.4 circ.m

This is the driver script that runs the entire program:

```
%filename: LV_sa.m
close all; clear all %clear all variables
clf      %and figures
global T TS tauS tauD;
global Csa Rs RMi RAo dt CHECK PLA;
in_LV_sa %initialize
t_heart = dt;
t_start = .2; %time exercise starts
t_end = .5;%time exercise stops
for klok=1:klokmax
   t=klok*dt;
  if t>t_start && t<t_end %start exercising
      Rs = 9; % ~Rs/2 for exercise
      Psa_set = 150; %raise the set pressure to ensure BP rises too
  end
   if t>t_end %stop exercising
      Psa_set = Psa_mean; %reset the set pressure back to the mean
   end
   if t>t_end+tauF
      Rs = 17.5; %reset the systemic resistance
      Psa_set = Psa_mean;
   end
  PLV_old=PLV;
  Psa_old=Psa;
  Qs_old = Qs;
  P_filt_old = P_filt;
  Q_filt_old = Q_filt;
  %Forward Euler Low Pass Filter:
  P_filt = (dt/tauF)*(Psa_old - P_filt_old) + P_filt_old; %Filtered Pressure
  Q_filt = (dt/tauF2)*(Qs_old - Q_filt_old) + Q_filt_old; %Filtered Flow
```

```
  %Change in heartrate based on pressure:
  deltaF = c*(F_heart*(P_filt - Psa_set)/Psa_set);
  F = F_heart - deltaF ;
  %Forward Euler to solve t_heart:
  t_heart_old = t_heart;
  t_heart = (F/F_heart)*dt + t_heart_old;

  CLV_old=CLV;
  CLV=CV_now(t_heart,CLVS,CLVD);
  %find self-consistent
  %valve states and pressures:
  set_SMi_SAo;

  %store in arrays for future plotting:
  t_plot(klok)=t;
  t_heart_plot(klok) = t_heart;
  CLV_plot(klok)=CLV;
  PLV_plot(klok)=PLV;
  Psa_plot(klok)=Psa;
  VLV_plot(klok)=CLV*PLV+VLVd;
  Vsa_plot(klok)=Csa*Psa+Vsad;
  QMi_plot(klok)=SMi*(PLA-PLV)/RMi;
  QAo_plot(klok)=SAo*(PLV-Psa)/RAo;
  Qs_plot(klok)=Psa/Rs;
  Qs = Qs_plot(klok);
  SMi_plot(klok)=SMi;
  SAo_plot(klok)=SAo;
  P_filt_plot(klok) = P_filt;
  Q_filt_plot(klok) = Q_filt;
  F_plot(klok) = F;
  F_heart_plot(klok) = F_heart;
end
```

6 Appendix: Parameter Initialization

Here we include all the parameter values that we used for the simulation:

```
%filename: in_LV_sa.m (initialization for LV_sa)
T =0.0125;   %Duration of heartbeat (minutes)
F_heart = 1/T; %constant
F = F_heart; %initial value for the heart rate variable ;
TS=0.0050;   %Duration of systole (minutes)
tauS=0.0025; %CLV time constant during systole (minutes)
tauD=0.0075; %CLV time constant during diastole (minutes)
Rs=17.86;    %Systemic resistance (mmHg/(liter/minute))
```

```
RMi=0.01;   %mitral valve resistance (mmHg/(liter/minute))
RAo=0.01;    %aortic valve resistance (mmHg/(liter/minute))

%The following value of Csa is approximate;
%needs adjustment to make blood pressure 120/80:

Csa=0.00175; %Systemic arterial compliance (liters/mmHg)
CLVS=0.00003; %Min (systolic) value of CLV (liters/mmHg)
CLVD=0.0146; %Max (diastolic) value of CLV (liters/mmHg)
Vsad=0.825 ; %Systemic arterial volume when Psa=0 (liters)
VLVd=0.027;  %Left ventricular volume when PLV=0 (liters)
PLA=5    ;   %Left atrial pressure (mmHg)
dt=0.01*T ;  %Time step duration (minutes)
%This choice implies 100 timesteps per cardiac cycle.

c=10; %gain of the feedback system (set to 0 to get uncontrolled model)



klokmax=80*T/dt; %Total number of timesteps
%simulation to run for 80 cardiac cycles
tmax = klokmax*dt;
PLV=5     ;            %Initial value of PLV (mmHg)
Psa=80    ;             %Initial value of Psa (mmHg)
P_filt = 93.470986390818837; %initial value of P_filtered (mmHg)
Qs = 5.6; %Initial value for cardiac output
Q_filt = 5.233537871826244; %initial value for filtered cardiac output
tauF = 4*T; %time constant of the low pass filter for P_filt
tauF2 = 4*T; %time constant of LPF of Q_filt
Psa_mean = 93.470986390818837;
Psa_set = Psa_mean; %set pressure starting at the mean

%set initial valve states:
SMi=(PLA>PLV); %evaluates to 1 if PLA>PLV, 0 otherwise
SAo=(PLV>Psa); %evaluates to 1 if PLV>Psa, 0 otherwise
CLV=CV_now(0,CLVS,CLVD); %Initial value of CLV (liters/mmHg)
%Initialize arrays used to store data for plotting:
%Although the program will work without doing this,
%it will run MUCH faster if MATLAB knows in advance
%how much space is needed for these arrays.
  t_plot=zeros(1,klokmax);
CLV_plot=zeros(1,klokmax);
PLV_plot=zeros(1,klokmax);
Psa_plot=zeros(1,klokmax);
VLV_plot=zeros(1,klokmax);
Vsa_plot=zeros(1,klokmax);
QMi_plot=zeros(1,klokmax);
```

```
QAo_plot=zeros(1,klokmax);
 Qs_plot=zeros(1,klokmax);
SMi_plot=zeros(1,klokmax);
SAo_plot=zeros(1,klokmax);
P_filt_plot=zeros(1,klokmax);
Q_filt_plot=zeros(1,klokmax);
%For self-checking in the function PLV_Psa_new, set CHECK=1
%To skip the self-checking, set CHECK=0
CHECK=1;
```

7 Conclusion

In this project, we modeled the effects of exercise on the human heart circulation controlled by the baroceptor feedback loop. We began by showing that the compartmental pulsatile model is not equipped to adjust to rapid changes in blood pressure and arterial dilation in the form of systemic resistance due to the fact that heartrate is a fixed parameter. In the new model, we treat heartrate as a varying function of time and show that we can achieve the effects of exercise in the pulsatile model by comparing a filtered signal of the pulsatile blood pressure to some set point and adjust the heart rate accordingly. We further improve on this by having the set point be a varying function of time.