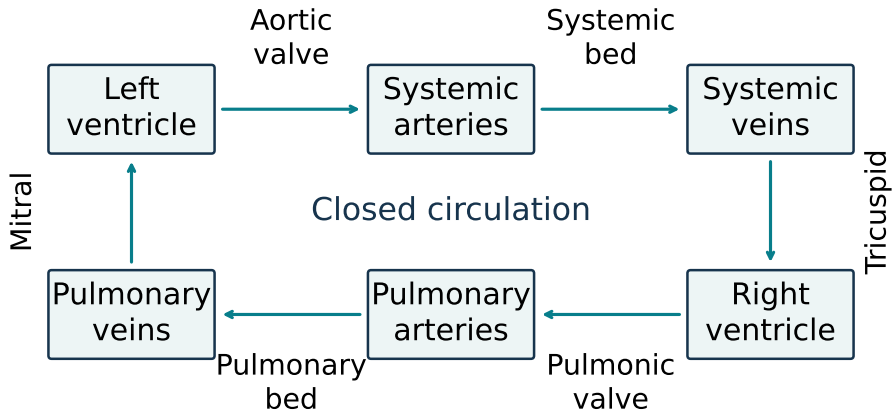
Computational Cardiology Lecture 4

Cardiac phase and pressure feedback in a pulsatile circulation

Zan Ahmad

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The two-pump circulation



Storage and boundary conditions

$$V_i = V_{d,i} + C_i(\phi)P_i.$$

The six-region companion is closed. The source feedback paper uses a ventricle and arterial storage with prescribed reservoirs.

A valve conducts only forward

$$q_{ij} = \frac{[P_i - P_j]_+}{R_{ij}}, \qquad [z]_+ = \max(z, 0).$$

Vascular beds remain open resistors with signed flow.

Contraction changes the pressure law

$$\frac{\mathrm{d}}{\mathrm{d}t}(CP) = C\dot{P} + \dot{C}P = q_{\mathrm{in}} - q_{\mathrm{out}}.$$

At fixed volume, decreasing C raises P.

The implicit hydraulic step

$$(D_C^{n+1} + hL_{\text{active}})P^{n+1} = D_C^n P^n.$$

Valve labels must agree with the resulting new-pressure differences.

Filter the arterial pressure

$$\tau \frac{\mathrm{d}\bar{P}}{\mathrm{d}t} = P_{sa} - \bar{P}.$$

The time constant separates the controller input from the rapid pressure pulse.

The filter update

$$\bar{P}^{n+1} = \bar{P}^n + \frac{h}{\tau}(P_{sa}^n - \bar{P}^n).$$

For $0 \leq h/\tau \leq 1$, the update lies between the two old pressures.

Bounded proportional rate control

$$F = \mathrm{clip}\left(F_0 \left[1 + g \frac{P_* - \bar{P}}{P_*} \right], F_{\min}, F_{\max} \right).$$

Low filtered pressure increases the requested heart rate.

A continuously varying clock

$$\dot{\phi} = F(t), \qquad \phi(t) = \phi(0) + \int_0^t F(s)\,\mathrm{d}s.$$

The compliance sequence is unchanged; the speed through it varies.

Why phase is not current rate times time

$$\frac{\mathrm{d}}{\mathrm{d}t}[tF(t)] = F(t) + t\dot{F}(t).$$

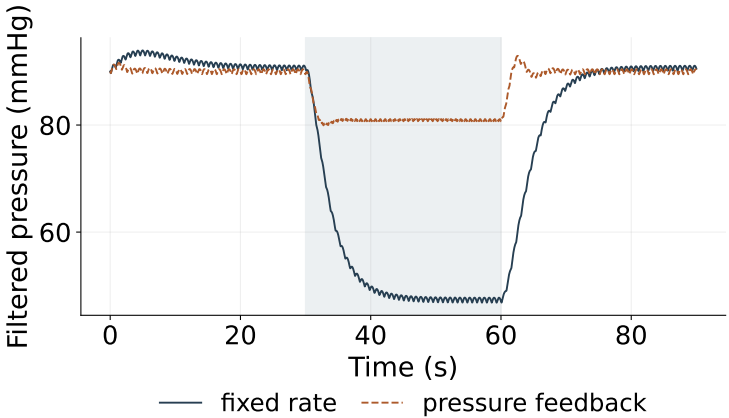
If $F(t) = F_0 + at$, correct phase is $F_0 t + at^2/2$, not $F_0 t + at^2$.

One phase for both ventricles

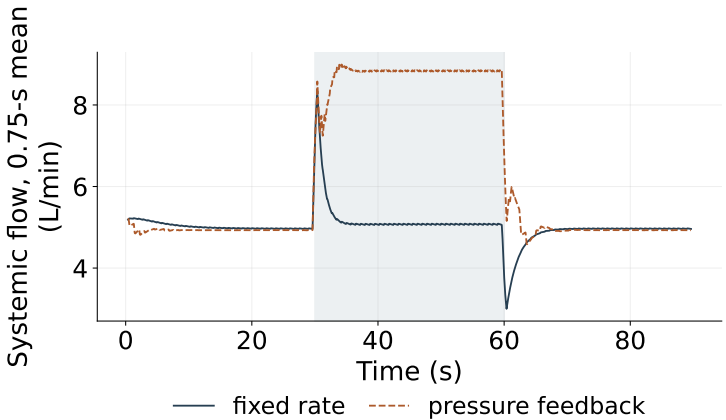
$$\phi^{n+1} = \phi^n + \frac{h_s}{60} F^{n+1}.$$

Both ventricular compliance laws use this accumulated phase. Seconds and minutes remain distinct.

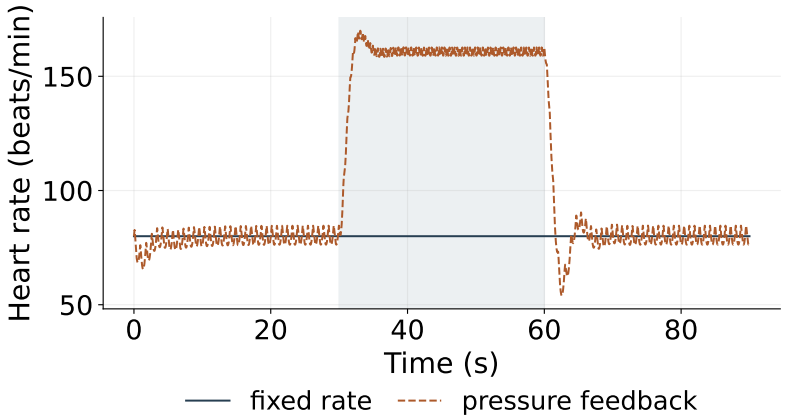
Arterial pressure under feedback



Systemic flow under feedback



Heart rate under feedback



A resistance disturbance

$$R_s : 17.86 \longrightarrow 8.93 \longrightarrow 17.86 \quad \mathrm{mmHg\,min/L}.$$

The reductions begin at 30 s and end at 60 s in the supplied experiment.

The required mean-flow change

$$\overline{Q}_s \simeq \frac{\Delta \overline{P}_s}{R_s}.$$

At a fixed pressure drop, halving resistance requires twice the flow. The coupled pressures need not remain fixed.

The proportional-control offset

$$\frac{P_* - \bar{P}}{P_*} = \frac{F_{\mathrm{req}}/F_0 - 1}{g}.$$

A rate above baseline requires a pressure shortfall when the target and gain are fixed and clipping is inactive.

Gain and delayed response

The pressure filter and blood redistribution delay the rate response. A larger gain reduces the static offset while potentially increasing oscillatory transient behavior.

Quantities retained by this controller

Rate control changes cycle timing. It does not independently regulate contractility, vascular tone or venous return. A centered plotting average is not the causal pressure filter used by the controller.