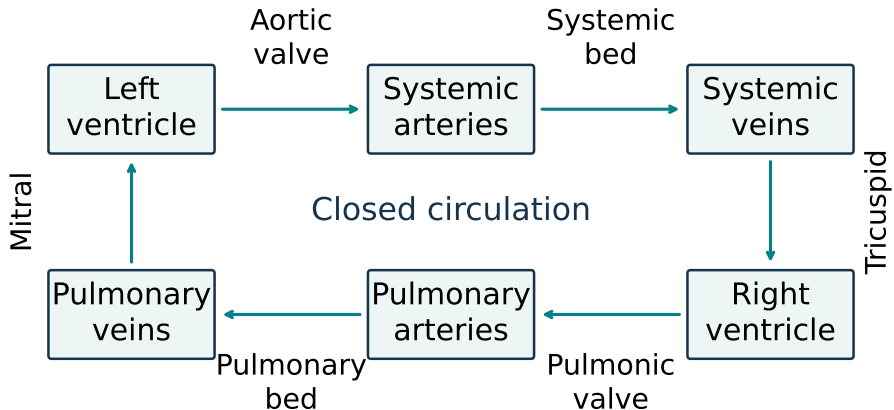
Computational Cardiology Lecture 4

# Cardiac phase and pressure feedback in a pulsatile circulation

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# The two-pump circulation



# Storage and boundary conditions

$$V_i = V_{d,i} + C_i(\phi)P_i.$$

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The six-region companion is closed. The source feedback paper uses a ventricle and arterial storage with prescribed reservoirs.

# A valve conducts only forward

$$q_{ij} = \frac{[P_i - P_j]_+}{R_{ij}}, \qquad [z]_+ = \max(z, 0).$$

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Vascular beds remain open resistors with signed flow.

# Contraction changes the pressure law

$$\frac{\mathrm{d}}{\mathrm{d}t}(CP) = C\dot{P} + \dot{C}P = q_{\mathrm{in}} - q_{\mathrm{out}}.$$

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At fixed volume, decreasing $C$ raises $P$.

# The implicit hydraulic step

$$(D_C^{n+1} + hL_{\text{active}})P^{n+1} = D_C^n P^n.$$

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Valve labels must agree with the resulting new-pressure differences.

# Filter the arterial pressure

$$\tau \frac{\mathrm{d}\bar{P}}{\mathrm{d}t} = P_{sa} - \bar{P}.$$

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The time constant separates the controller input from the rapid pressure pulse.

# The filter update

$$\bar{P}^{n+1} = \bar{P}^n + \frac{h}{\tau}(P_{sa}^n - \bar{P}^n).$$

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For $0 \leq h/\tau \leq 1$, the update lies between the two old pressures.

# Bounded proportional rate control

$$F = \mathrm{clip}\left( F_0 \left[ 1 + g \frac{P_* - \bar{P}}{P_*} \right], F_{\min}, F_{\max} \right).$$

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Low filtered pressure increases the requested heart rate.

# A continuously varying clock

$$\dot{\phi} = F(t), \qquad \phi(t) = \phi(0) + \int_0^t F(s)\,\mathrm{d}s.$$

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The compliance sequence is unchanged; the speed through it varies.

# Why phase is not current rate times time

$$\frac{\mathrm{d}}{\mathrm{d}t}[tF(t)] = F(t) + t\dot{F}(t).$$

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$$\frac{\mathrm{d}}{\mathrm{d}t}[tF(t)] = F(t) + t\dot{F}(t).$$

If $F(t) = F_0 + at$, correct phase is $F_0 t + at^2/2$, not $F_0 t + at^2$.

# One phase for both ventricles

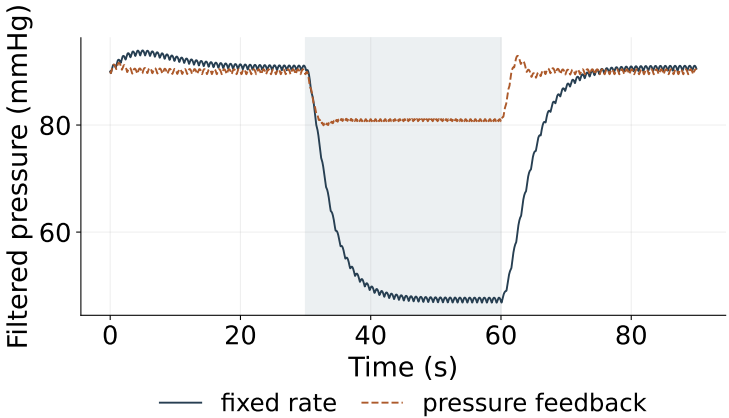
$$\phi^{n+1} = \phi^n + \frac{h_s}{60} F^{n+1}.$$

# One phase for both ventricles

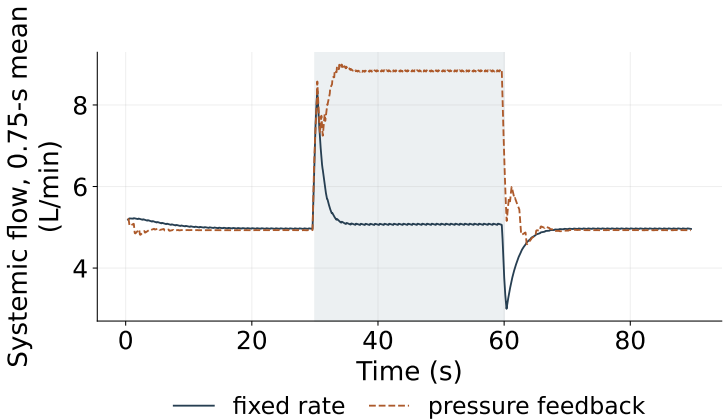
$$\phi^{n+1} = \phi^n + \frac{h_s}{60} F^{n+1}.$$

Both ventricular compliance laws use this accumulated phase. Seconds and minutes remain distinct.

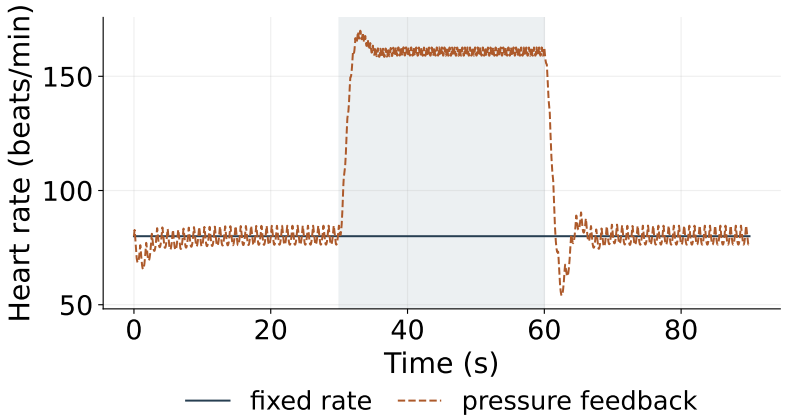
# Arterial pressure under feedback



# Systemic flow under feedback



# Heart rate under feedback



# A resistance disturbance

$$R_s : 17.86 \longrightarrow 8.93 \longrightarrow 17.86 \quad \text{mmHg min/L}.$$

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The reductions begin at $30$ s and end at $60$ s in the supplied experiment.

# The required mean-flow change

$$\overline{Q}_s \simeq \frac{\Delta \overline{P}_s}{R_s}.$$

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At a fixed pressure drop, halving resistance requires twice the flow. The coupled pressures need not remain fixed.

# The proportional-control offset

$$\frac{P_* - \bar{P}}{P_*} = \frac{F_{\mathrm{req}}/F_0 - 1}{g}.$$

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A rate above baseline requires a pressure shortfall when the target and gain are fixed and clipping is inactive.

# Gain and delayed response

The pressure filter and blood redistribution delay the rate response.

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# Quantities retained by this controller

Rate control changes cycle timing. It does not independently regulate contractility, vascular tone or venous return.

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Rate control changes cycle timing. It does not independently regulate contractility, vascular tone or venous return. A centered plotting average is not the causal pressure filter used by the controller.