Computational Cardiology Lecture 12

Local convolutional segmentation and geometric error

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The output of segmentation

$$I \longmapsto \widehat{M} \in \{0,1\}^{H \times W}.$$

A spatial mask contains boundary and connectivity information absent from one image-level label.

A local shared predictor

$$z_i = w^{\mathsf{T}} x_i + b, \qquad p_i = \sigma(z_i).$$

The 3×3 patch supplies nine weights and one shared bias.

The sigmoid decision

$$\sigma(z) = \frac{1}{1+e^{-z}}, \qquad \widehat{y}_i = \mathbf{1}_{\{p_i \geq .5\}}.$$

With threshold $.5$, the class boundary is $z_i = 0$.

Receptive field and padding

The rule uses only a 3×3 neighborhood. Reflect padding supplies border context. A distant anatomical relationship lies outside this receptive field.

Binary cross-entropy from logits

$$J = \frac{1}{M} \sum_i [\log(1 + e^{z_i}) - y_i z_i] + \frac{\lambda}{2} \|w\|^2.$$

A stable log-add-exp evaluation avoids taking logarithms of rounded probabilities.

Differentiate the data term

$$\frac{\partial \ell_i}{\partial z_i} = p_i - y_i.$$

$$\nabla_w J = \frac{1}{M} X^{\mathsf{T}}(p - y) + \lambda w, \quad \partial_b J = \frac{1}{M} \mathbf{1}^{\mathsf{T}}(p - y).$$

Curvature of the local model

$$\nabla^2 \ell = \frac{1}{M} \widetilde{X}^{\mathsf{T}} \operatorname{diag}(p_i(1-p_i)) \widetilde{X} \succeq 0.$$

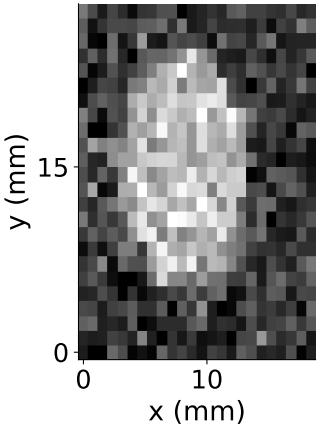
The single-layer objective is convex; weight sharing does not make it a deep network.

A gradient update

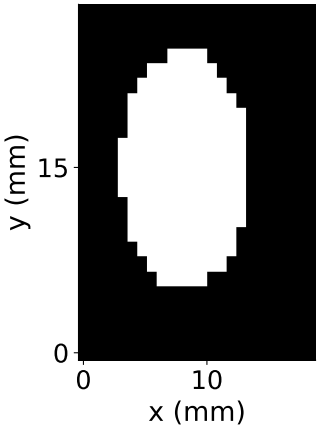
$$w^{k+1} = w^k - \eta \nabla_w J, \qquad b^{k+1} = b^k - \eta \partial_b J.$$

Validation chooses among saved iterates without using the test set.

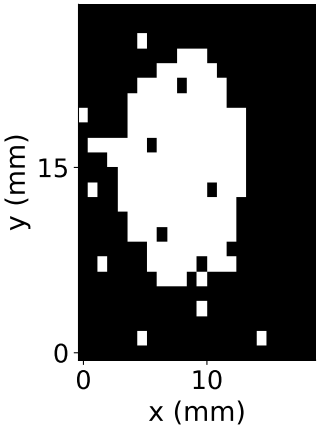
Synthetic test image



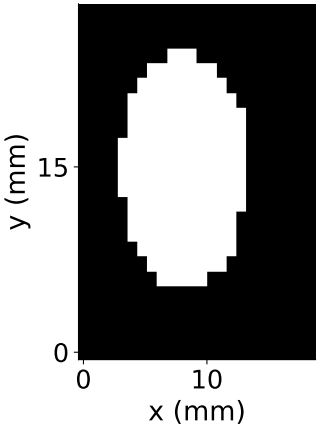
Generator reference



Fixed threshold 0.5



Learned 3×3 + sigmoid



The unit of separation

Complete generated images remain in one split. Neighboring patches from one image share both pixels and geometry; splitting pixels would answer an easier prediction problem.

Overlap as a set quantity

$$\mathrm{Dice}(P, R) = \frac{2|P \cap R|}{|P| + |R|}.$$

Equal overlap need not imply equal physical boundary error.

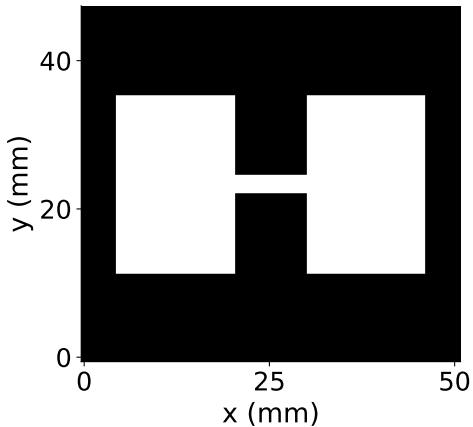
Physical boundary distance

Compute nearest-boundary distances in both directions using physical coordinates. Average distance describes typical displacement; Hausdorff distance records the largest nearest-boundary discrepancy.

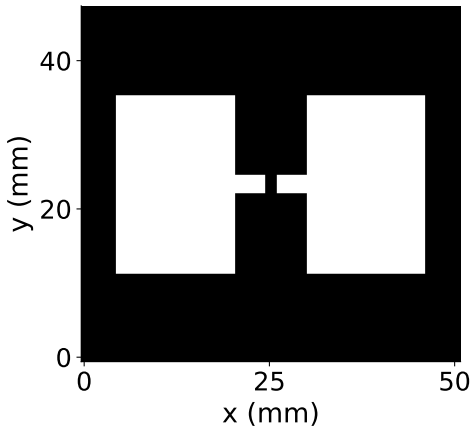
Empty-set conventions

Two empty masks have Dice one in this implementation. One empty mask has Dice zero. Boundary distance is unavailable when only one boundary is empty.

Reference: 1 component



Four missing pixels: 2 components



A nearly perfect overlap can sever a bridge

$$|R| = 824, \quad |P| = 820, \quad \text{Dice} = \frac{1640}{1644} \approx .99757.$$

Four deleted bridge pixels change one connected component into two.

Area bias and overlap are distinct

$$e_A = \frac{A_P - A_R}{A_R}.$$

A signed mean can conceal cancellation of over- and under-segmentation across cases.

The downstream geometric requirement

A transport model depends on the paths and boundaries of its domain. An overlap score cannot by itself certify preservation of those paths.