Computational Cardiology Lecture 16

Probability calibration and missing-modality evaluation

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Three properties of a score

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Aggregate repeated observations

$$p_g = \sigma((z_{g1} + z_{g2})/2).$$

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This is generally different from averaging the two probabilities.

The paired modality comparison

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Represent a missing measurement

$$B^* = \begin{cases} B, & \text{observed,} \\ m_B^{train}, & \text{missing,} \end{cases} \qquad r_B = \mathbf{1}_{\{B \text{ missing}\}}.$$

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The indicator distinguishes imputation from an observed median value.

Freeze preprocessing

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Temperature-scaled probabilities

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The supplied experiment first sharpens logits by a declared factor 2.5.

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Positive temperature preserves ranking and ties, hence the pairwise AUC.

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Other fixed probability thresholds need not preserve their classifications.

Fit temperature using validation likelihood

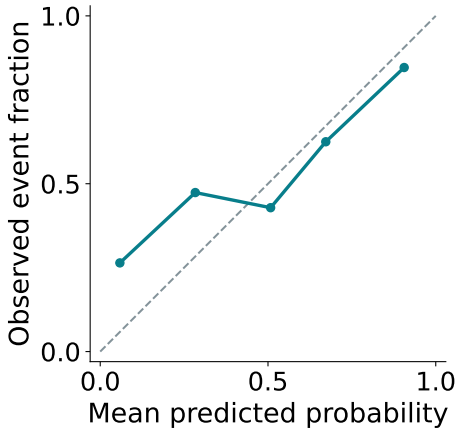
$$\widehat{T} = \arg\min_{T>0} \frac{1}{N} \sum_i [\log(1 + e^{z_i/T}) - y_i z_i / T].$$

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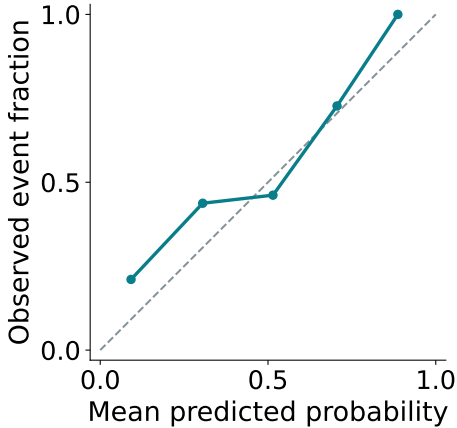
$$\widehat{T} = \arg\min_{T>0} \frac{1}{N} \sum_i [\log(1 + e^{z_i/T}) - y_i z_i / T].$$

The base model and all preprocessing remain fixed.

Sharpened AB scores



Validation-calibrated AB



The Brier score

$$\text{Brier} = \frac{1}{N}\sum_i (p_i - y_i)^2.$$

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It evaluates probabilities, not only the chosen classification threshold.

Conditional probability decomposition

$$\mathbb{E}[(p - Y)^2 \mid X] = (p - \eta)^2 + \eta(1 - \eta).$$

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The minimum occurs at the true conditional event probability η.

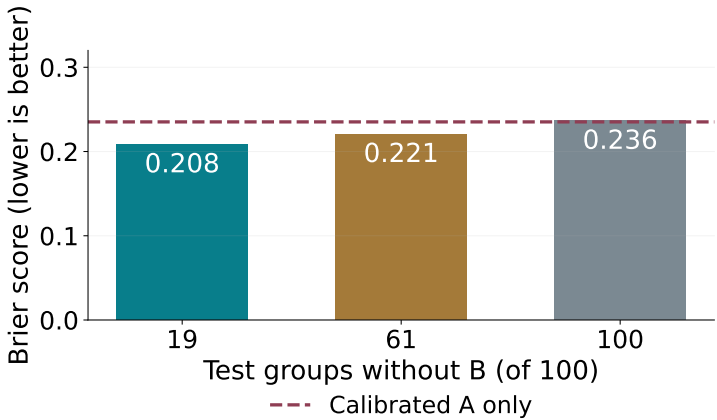
Reliability bins

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Brier score (lower is better)



Remove B while retaining the same groups

Replace B by the frozen missing-data rule and retain every test outcome.

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AUC and probability quality can separate

Positive temperature leaves AUC unchanged while changing Brier score and likelihood.

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Distribution-dependent calibration

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can invalidate a fitted calibration map even when the implementation is unchanged.