Computational Cardiology Lecture 18

# Spectral attenuation surrogates and graph-basis limitations

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# A three-parameter fixed-time map

$$\widehat{\Phi}_T[u] = a_0 + \sum_{k=1}^{3} q_k a_k \cos(k\pi x/L).$$

The mean passes through unchanged; three gains are fitted.

# Resolved midpoint projection

$$a_0 = \frac{1}{N}\sum_i u_i, \qquad a_k = \frac{2}{N}\sum_i u_i \cos(k\pi x_i/L).$$

Exact coefficient recovery here relies on the resolved finite cosine family.

# Projection and reconstruction

$$u \longmapsto (a_0, a_1, a_2, a_3) \longmapsto (a_0, q_1 a_1, q_2 a_2, q_3 a_3) \longmapsto \widehat{u}.$$

Omitted input modes cannot reappear through this diagonal map.

# A graph-normalized Laplacian

$$L_{sym} = I - D^{-1/2} W D^{-1/2}.$$

Assume positive degrees. Its eigenvectors use the Euclidean inner product of the normalized representation.

# The normalized null mode

$$L_{sym} D^{1/2} \mathbf{1} = 0.$$

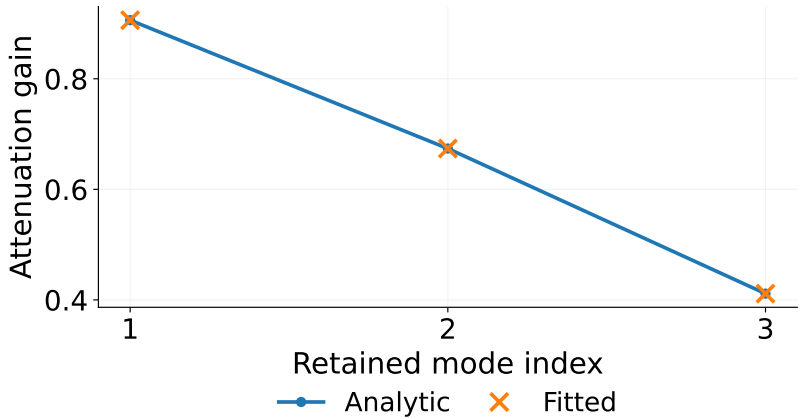
The zero mode is proportional to square-root degree, generally not the constant vector.

# Least squares for one gain

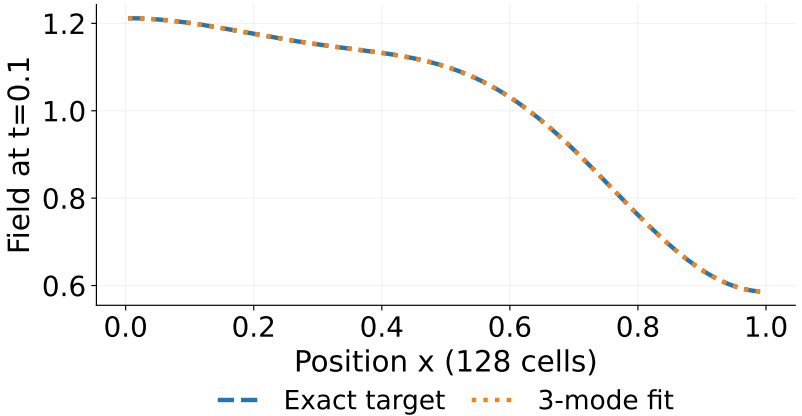
$$J(q) = \sum_j (b_j - q a_j)^2.$$

$$J'(q) = -2 \sum_j a_j b_j + 2q \sum_j a_j^2.$$

# An exactly representable fit



# Reuse on a finer sampling grid



# The fitted attenuation

$$\widehat{q} = \frac{\sum_j a_j b_j}{\sum_j a_j^2}.$$

A positive denominator gives a unique minimizer.

# An unexcited mode is unidentified

If $a_j = 0$ for every case, $J(q) = \sum_j b_j^2$ for every $q$. No optimizer can recover a gain from absent excitation.

# Exact representability explains the fit

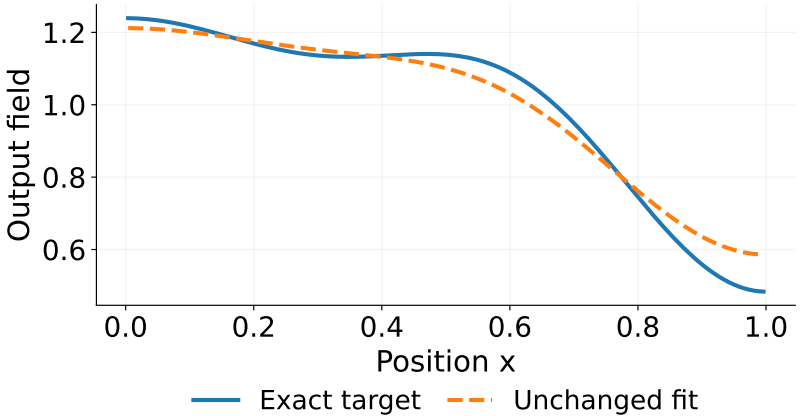
$$q_k = e^{-\kappa(k\pi/L)^2 T}.$$

Noiseless exact labels, a known basis and three excited modes make near-roundoff error expected in the stated family.

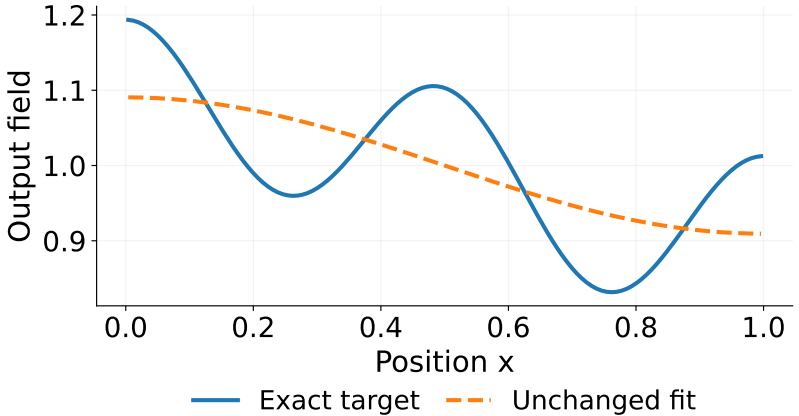
# Reuse on another resolved grid

The same continuous coefficients are projected and reconstructed on a new midpoint grid. This changes sampling density while retaining the domain, bandwidth, diffusivity and time.

# Changed diffusivity



# An omitted input mode



# An omitted fourth mode

$$E_{RMS} = \frac{|a_4|}{\sqrt{2}} e^{-\kappa(4\pi/L)^2 T}.$$

The mean can remain exact while this nonconstant output is missing.

# A changed diffusivity

$$e^{-\kappa_{train}(k\pi/L)^2 T} \neq e^{-\kappa_{test}(k\pi/L)^2 T}.$$

Keeping the basis does not correct gains fitted for another operator.

# Consistent eigenvector sign changes

$$K = \Phi Q \Phi^{\mathsf{T}} = (\Phi S) Q (\Phi S)^{\mathsf{T}}, \qquad S^2 = I.$$

Diagonal gains commute with sign flips.

# Repeated eigenspaces require more

$$Q \longmapsto OQO^{\mathsf{T}}.$$

Invariance to every orthogonal change within a block requires identical gains on that block.

# The research scope remains broader

GFuNK learns graph-spectral nonlinear generators and integrates trajectories across specified geometry and diffusion families. The three-gain companion is a fixed-time baseline, not that architecture.