Computational Cardiology Lecture 18

Spectral attenuation surrogates and graph-basis limitations

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A three-parameter fixed-time map

$$\widehat{\Phi}_T[u] = a_0 + \sum_{k=1}^{3} q_k a_k \cos(k\pi x/L).$$

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The mean passes through unchanged; three gains are fitted.

Resolved midpoint projection

$$a_0 = \frac{1}{N}\sum_i u_i, \qquad a_k = \frac{2}{N}\sum_i u_i \cos(k\pi x_i/L).$$

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Exact coefficient recovery here relies on the resolved finite cosine family.

Projection and reconstruction

$$u \longmapsto (a_0, a_1, a_2, a_3) \longmapsto (a_0, q_1 a_1, q_2 a_2, q_3 a_3) \longmapsto \widehat{u}.$$

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Omitted input modes cannot reappear through this diagonal map.

A graph-normalized Laplacian

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Assume positive degrees. Its eigenvectors use the Euclidean inner product of the normalized representation.

The normalized null mode

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The zero mode is proportional to square-root degree, generally not the constant vector.

Least squares for one gain

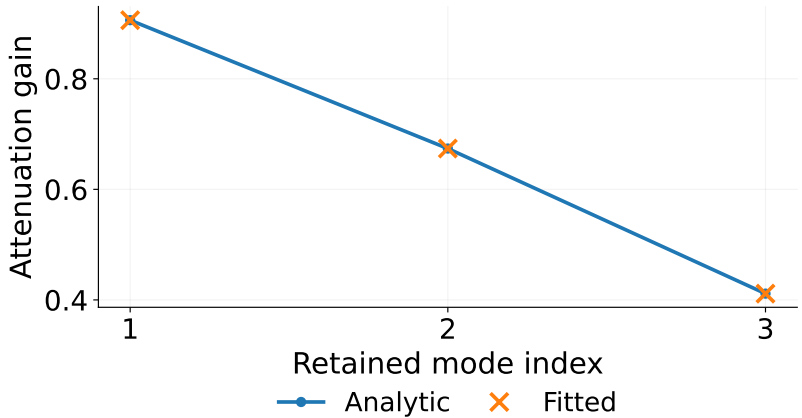
$$J(q) = \sum_j (b_j - q a_j)^2.$$

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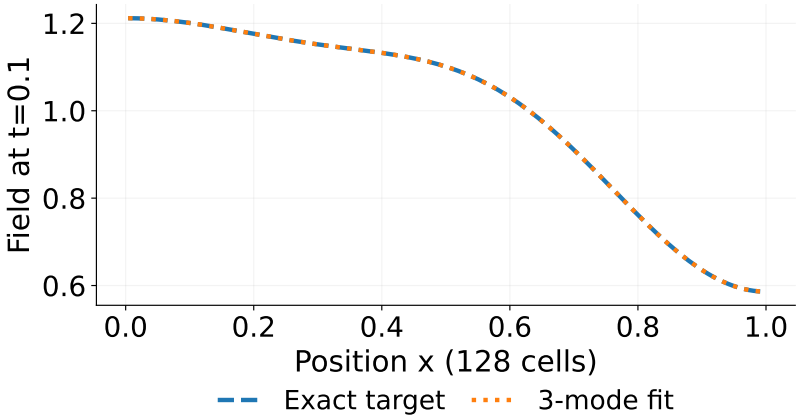
$$J(q) = \sum_j (b_j - q a_j)^2.$$

$$J'(q) = -2 \sum_j a_j b_j + 2q \sum_j a_j^2.$$

An exactly representable fit



Reuse on a finer sampling grid



The fitted attenuation

$$\widehat{q} = \frac{\sum_j a_j b_j}{\sum_j a_j^2}.$$

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A positive denominator gives a unique minimizer.

An unexcited mode is unidentified

If $a_j = 0$ for every case, $J(q) = \sum_j b_j^2$ for every q.

An unexcited mode is unidentified

If $a_j = 0$ for every case, $J(q) = \sum_j b_j^2$ for every q. No optimizer can recover a gain from absent excitation.

Exact representability explains the fit

$$q_k = e^{-\kappa(k\pi/L)^2 T}.$$

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Noiseless exact labels, a known basis and three excited modes make near-roundoff error expected in the stated family.

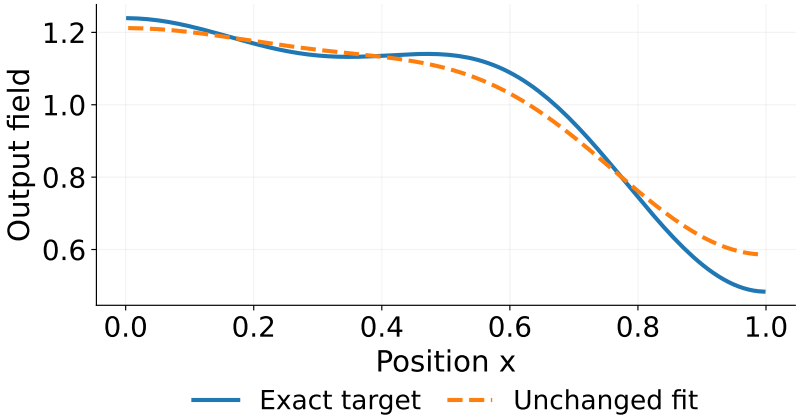
Reuse on another resolved grid

The same continuous coefficients are projected and reconstructed on a new midpoint grid.

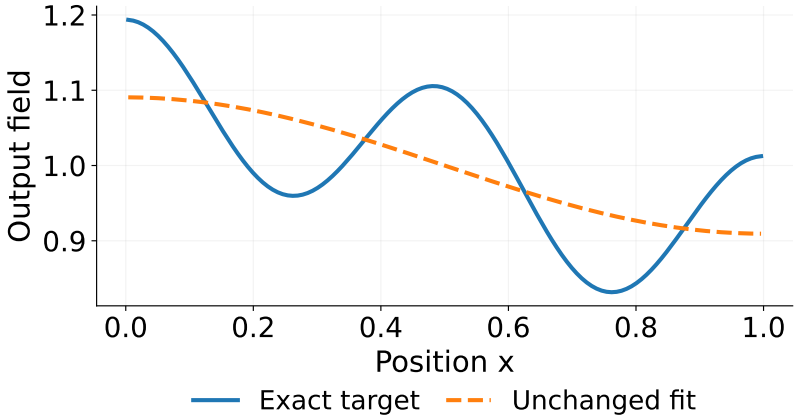
Reuse on another resolved grid

The same continuous coefficients are projected and reconstructed on a new midpoint grid. This changes sampling density while retaining the domain, bandwidth, diffusivity and time.

Changed diffusivity



An omitted input mode



An omitted fourth mode

$$E_{RMS} = \frac{|a_4|}{\sqrt{2}} e^{-\kappa(4\pi/L)^2 T}.$$

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The mean can remain exact while this nonconstant output is missing.

A changed diffusivity

$$e^{-\kappa_{train}(k\pi/L)^2 T} \neq e^{-\kappa_{test}(k\pi/L)^2 T}.$$

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Keeping the basis does not correct gains fitted for another operator.

Consistent eigenvector sign changes

$$K = \Phi Q \Phi^{\mathsf{T}} = (\Phi S) Q (\Phi S)^{\mathsf{T}}, \qquad S^2 = I.$$

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Diagonal gains commute with sign flips.

Repeated eigenspaces require more

$$Q \longmapsto OQO^{\mathsf{T}}.$$

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Invariance to every orthogonal change within a block requires identical gains on that block.

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GFuNK learns graph-spectral nonlinear generators and integrates trajectories across specified geometry and diffusion families.

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GFuNK learns graph-spectral nonlinear generators and integrates trajectories across specified geometry and diffusion families. The three-gain companion is a fixed-time baseline, not that architecture.