Computational Cardiology Lecture 19

Surrogate error, amortized cost and training-step bias

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Three solutions in an error budget

$$\|u_{sur} - u\| \leq \|u_{sur} - u_{ref}\| + \|u_{ref} - u\|.$$

Surrogate agreement and reference accuracy are separate terms.

Compatibility of the comparison

The same field, physical time, coordinate representation and norm must be used. The bound does not estimate disagreement with biological observations.

Total reference cost

$$C_{ref}(Q) = QC_r.$$

Q counts comparable queries under a declared timing boundary.

Total surrogate cost

$$C_{sur}(Q) = C_0 + QC_s.$$

Setup can include labels, preprocessing, spectral bases, fitting and model selection.

Strict break-even

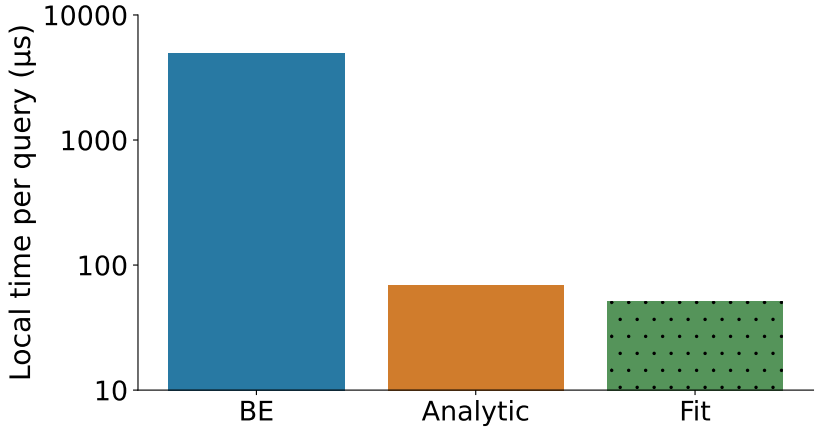
$$Q > \frac{C_0}{C_r - C_s}, \qquad C_r > C_s.$$

The first strictly cheaper integer count is the floor of this ratio plus one.

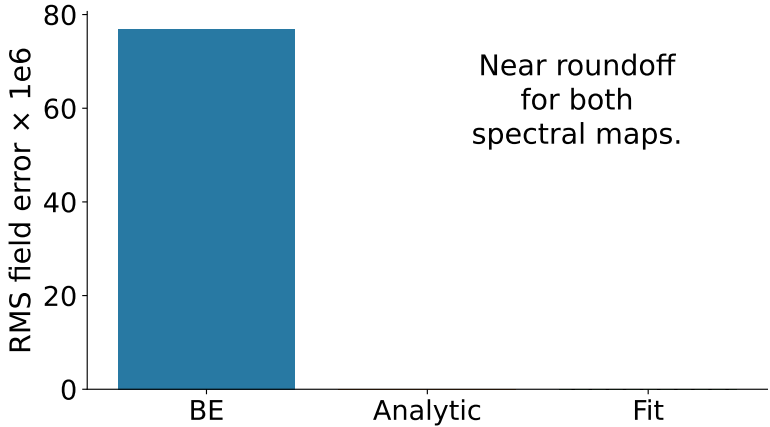
Geometry-dependent setup

A basis constructed for each geometry is a recurring cost when geometries change. Treating it as one universal setup changes the workload being compared.

Measured CPU cost



Error against analytic truth



A cost–error comparison

Compare admissible pairs (C, E) rather than cost alone. On the heat fixture, analytic spectral evolution is a competing baseline.

The exact scalar flow

$$y_t = -\lambda y, \qquad y(t+\Delta) = e^{-\lambda\Delta} y(t).$$

Take $y(0) = 1$ for the numerical example.

Fit a generator through forward Euler

$$y^{n+1} = (1 + \Delta a_\Delta) y^n.$$

Matching exact data gives

$$a_\Delta = \frac{e^{-\lambda\Delta} - 1}{\Delta}.$$

The fitted rate differs from the true rate

$$a_\Delta = -\lambda + \frac{\lambda^2}{2}\Delta + O(\Delta^2).$$

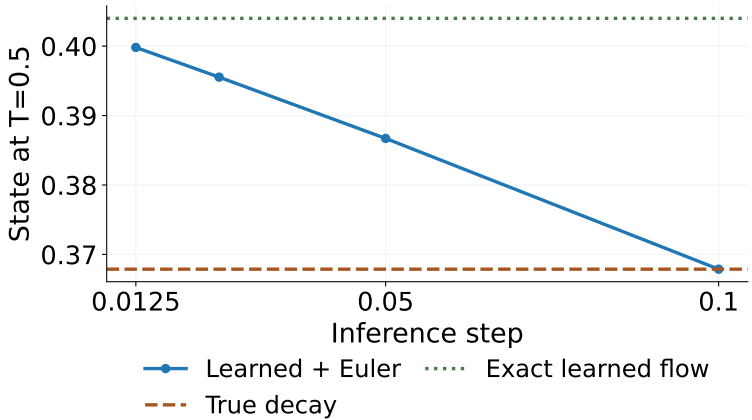
A finite training step is encoded in the fitted generator.

Exact sampled agreement at the training step

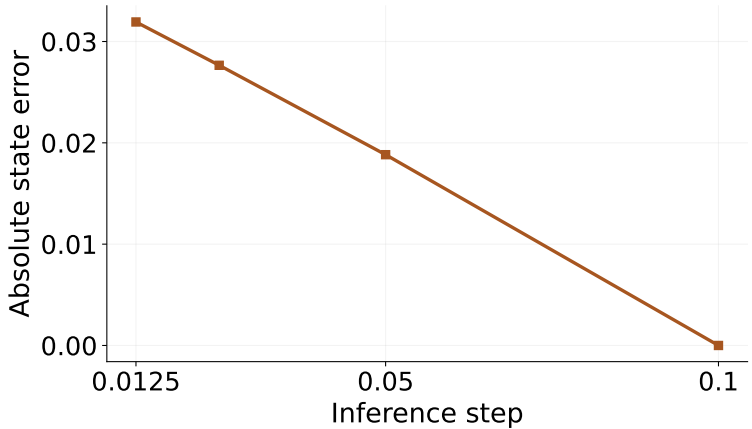
$$(1 + \Delta a_{\Delta})^n = e^{-\lambda n \Delta}.$$

This identity follows from the matching construction, not from recovering $-\lambda$.

A step-coupled fitted generator



Smaller steps need not help



Refine inference while holding the fit fixed

$$(1 + h a_\Delta)^{T/h} \longrightarrow e^{T a_\Delta} \quad (h \to 0).$$

The limit solves the learned equation more accurately.

A nonvanishing limiting bias

For $\lambda = 2$, $\Delta = .1$, $T = .5$,

$$e^{Ta_\Delta} \approx .403998, \qquad e^{-\lambda T} \approx .367879.$$

A smaller inference step need not reduce error to the true equation.

Two distinct refinement limits

$$h \to 0 \quad \text{at fixed } \Delta \qquad \text{versus} \qquad \Delta \to 0.$$

The first refines integration; the second changes the fitted generator.

An admissible computational replacement

The surrogate must meet the stated error tolerance over the intended input regime and query workload. Lower per-query cost alone does not establish that claim.