Computational Cardiology Lecture 19

Surrogate error, amortized cost and training-step bias

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Three solutions in an error budget

$$\|u_{sur} - u\| \leq \|u_{sur} - u_{ref}\| + \|u_{ref} - u\|.$$

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Surrogate agreement and reference accuracy are separate terms.

Compatibility of the comparison

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Total reference cost

$$C_{ref}(Q) = QC_r.$$

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Q counts comparable queries under a declared timing boundary.

Total surrogate cost

$$C_{sur}(Q) = C_0 + QC_s.$$

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Setup can include labels, preprocessing, spectral bases, fitting and model selection.

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The first strictly cheaper integer count is the floor of this ratio plus one.

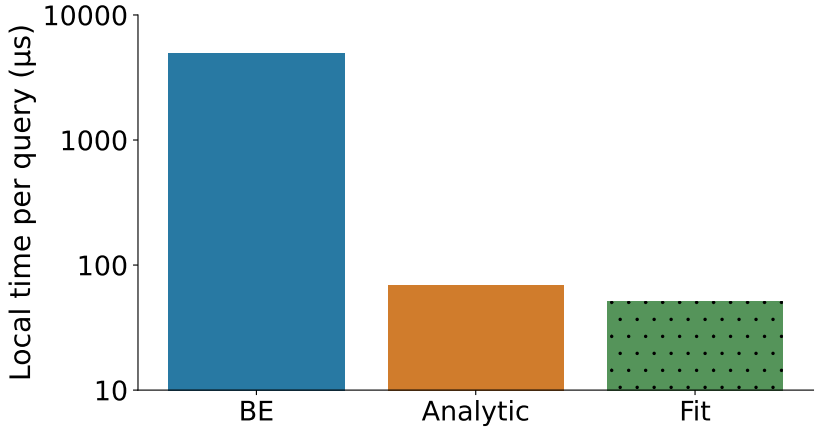
Geometry-dependent setup

A basis constructed for each geometry is a recurring cost when geometries change.

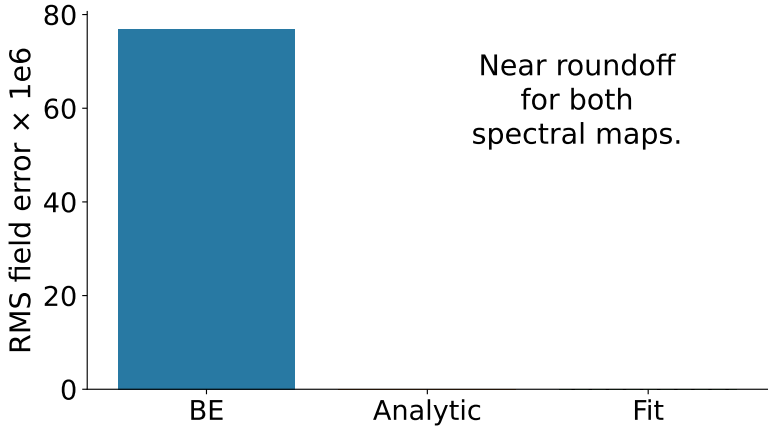
Geometry-dependent setup

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Measured CPU cost



Error against analytic truth



A cost–error comparison

Compare admissible pairs (C, E) rather than cost alone.

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Compare admissible pairs (C, E) rather than cost alone. On the heat fixture, analytic spectral evolution is a competing baseline.

The exact scalar flow

$$y_t = -\lambda y, \qquad y(t+\Delta) = e^{-\lambda\Delta} y(t).$$

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Take $y(0) = 1$ for the numerical example.

Fit a generator through forward Euler

$$y^{n+1} = (1 + \Delta a_{\Delta}) y^{n}.$$

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Matching exact data gives

$$a_\Delta = \frac{e^{-\lambda\Delta} - 1}{\Delta}.$$

The fitted rate differs from the true rate

$$a_\Delta = -\lambda + \frac{\lambda^2}{2}\Delta + O(\Delta^2).$$

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A finite training step is encoded in the fitted generator.

Exact sampled agreement at the training step

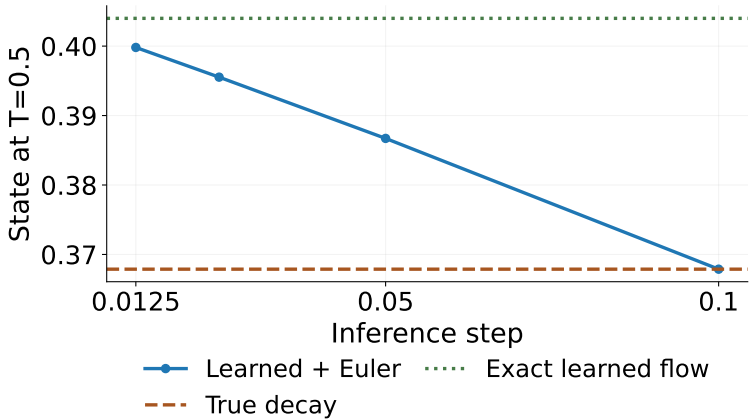
$$(1 + \Delta a_{\Delta})^n = e^{-\lambda n \Delta}.$$

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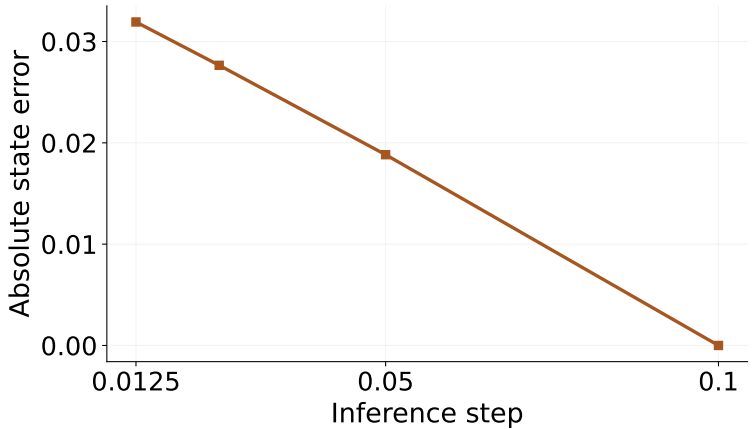
$$(1 + \Delta a_{\Delta})^n = e^{-\lambda n \Delta}.$$

This identity follows from the matching construction, not from recovering $-\lambda$.

A step-coupled fitted generator



Smaller steps need not help



Refine inference while holding the fit fixed

$$(1 + h a_\Delta)^{T/h} \longrightarrow e^{T a_\Delta} \quad (h \to 0).$$

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The limit solves the learned equation more accurately.

A nonvanishing limiting bias

For $\lambda = 2$, $\Delta = .1$, $T = .5$,

$$e^{Ta_\Delta} \approx .403998, \qquad e^{-\lambda T} \approx .367879.$$

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A smaller inference step need not reduce error to the true equation.

Two distinct refinement limits

$$h \to 0 \quad \text{at fixed } \Delta \qquad \text{versus} \qquad \Delta \to 0.$$

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The first refines integration; the second changes the fitted generator.

An admissible computational replacement

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