Computational Cardiology Lecture 20

Field errors, conserved moments and correlation invariance

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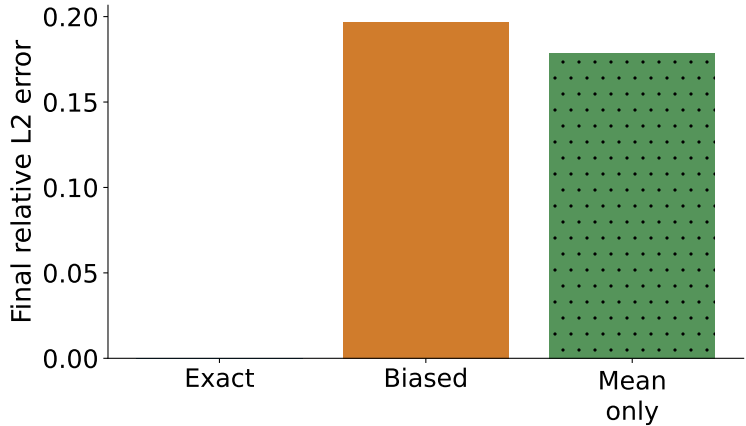
An observable selects part of a solution

Mean flow, total volume, arrival time and correlation summarize different features.

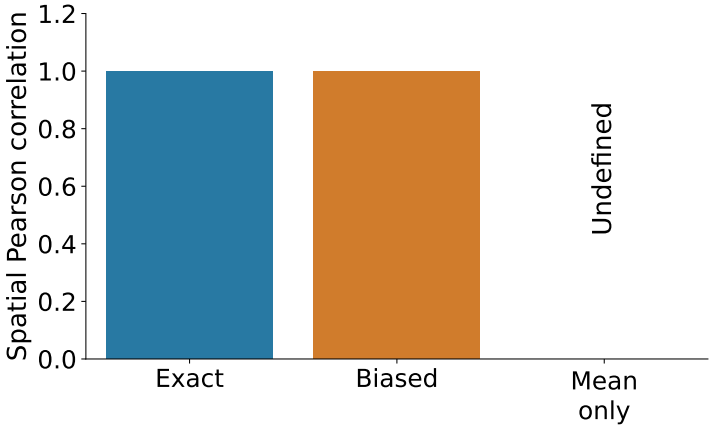
An observable selects part of a solution

Mean flow, total volume, arrival time and correlation summarize different features. Agreement in one does not determine the complete state or field.

A field metric sees both failures



Correlation can miss a bias



Two explicit candidate fields

$$p_{bias}(x,t) = u(x,t) + .2, \qquad p_{mean}(x,t) = \overline{u}(t).$$

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The exact sealed heat trajectory supplies a common reference.

Absolute RMS error

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A constant offset b gives $E_{RMS} = |b|$.

Relative error needs a nonzero reference norm

$$E_{rel} = \frac{\|p - y\|}{\|y\|}.$$

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If $\|y\| = 0$, this quotient is undefined without a separate convention.

Pearson correlation

$$\rho(p, y) = \frac{(p - \bar{p}\mathbf{1})^{\mathsf{T}}(y - \bar{y}\mathbf{1})}{\|p - \bar{p}\mathbf{1}\| \, \|y - \bar{y}\mathbf{1}\|}.$$

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Both centered norms must be nonzero.

Center an affine prediction

$$p = ay + b\mathbf{1} \quad \Longrightarrow \quad p - \bar{p}\mathbf{1} = a(y - \bar{y}\mathbf{1}).$$

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The offset disappears.

Correlation ignores positive rescaling

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Positive amplitude errors and arbitrary offsets can coexist with correlation one.

A hand calculation

$$y = (1, 3, 5), \qquad p = (3, 5, 7).$$

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Both centered vectors are $(-2, 0, 2)$, while the error is $(2, 2, 2)$.

Compatible but different conclusions

$$\rho = 1, \qquad E_{RMS} = 2, \qquad \bar{p} - \bar{y} = 2.$$

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These numbers answer different questions.

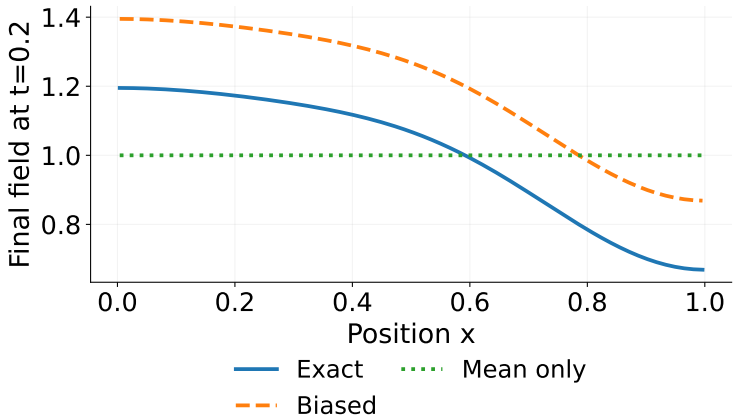
A constant prediction

If $p_i = \bar{y}$ for every i, its mean is correct but its centered norm is zero.

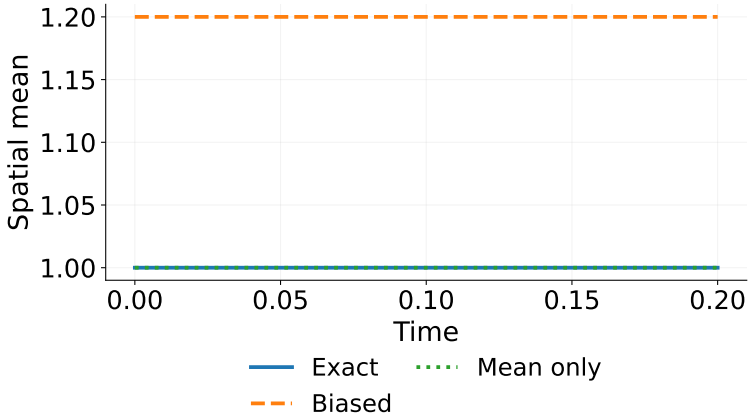
A constant prediction

If $p_i = \bar{y}$ for every i, its mean is correct but its centered norm is zero. Correlation is undefined and nonconstant reference structure is lost.

Three different field claims



Conserved does not mean correct



Conservation relative to the initial prediction

$$r_{drift}(t) = \overline{p}(t) - \overline{p}(0).$$

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An offset candidate can have zero drift throughout the trajectory.

Conservation for the prescribed problem

$$r_{prescribed}(t) = \overline{p}(t) - \overline{y}(0).$$

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$$r_{prescribed}(t) = \overline{p}(t) - \overline{y}(0).$$

For $p = y + b$, this residual equals b, exposing the incorrect initial mean.

Orthogonal decomposition of field error

$$e = \bar{e}\mathbf{1} + e_c, \qquad \mathbf{1}^{\mathsf{T}} e_c = 0.$$

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$$\frac{1}{N}\|e\|^2 = \bar{e}^2 + \frac{1}{N}\|e_c\|^2.$$

What the mean cannot determine

A mean constraint controls one component of the field error.

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A mean constraint controls one component of the field error. Every nonconstant mode can still be wrong, even when the mean history is exact.

A precise synthesis

Conservation constrains balances; alignment constrains allowed pose; arrival times constrain events; field norms constrain amplitudes.

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Conservation constrains balances; alignment constrains allowed pose; arrival times constrain events; field norms constrain amplitudes. The mathematical claim should match the observable actually established.