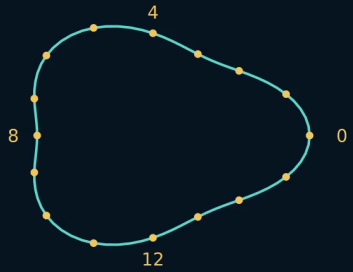
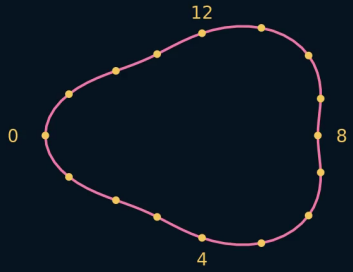
LANDMARK GEOMETRY

A distance requires a correspondence

Coordinates use a common frame and the same length unit.





$$d(X,Y) = \sqrt{\frac{1}{N}\sum_{i=1}^{N} \|x_i - y_i\|^2}$$

The lab can center, rotate and scale configurations. Each choice changes what the reported distance means.

COORDINATE INTERPOLATION

A half-turn collapses halfway

Let the target be Y = −X and preserve landmark labels.

$$F_t(x) = (1 - 2t)x$$



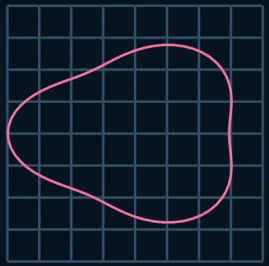
t = 0.50 det DF_t = 0.000

At t = 1/2 every point maps to the origin. A positive endpoint determinant does not protect the path.

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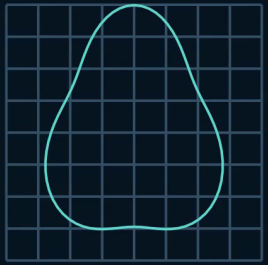
t = 1.00 det DF_t = 1.000

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AMBIENT FLOW

A smooth rotation reaches the same endpoint

The velocity field specifies motion throughout the ambient plane.



$$\dot{\Phi}_t = \pi J \Phi_t$$

$$\Phi_t = R_{\pi t}$$

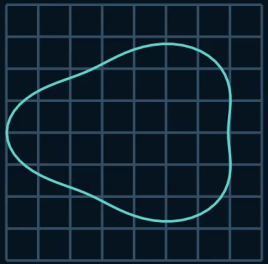
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J rotates a vector through 90°. Uniqueness of the smooth ODE prevents trajectory collisions.

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FLOW AND DEFORMATION

The Jacobian evolves with the divergence

A numerical integration still needs independent step-size and orientation checks.

$$\dot{\Phi}_t(x) = u(\Phi_t(x)), \qquad \Phi_0(x) = x$$

$$\frac{\mathrm{d}}{\mathrm{d}t} D\Phi_t = Du(\Phi_t) D\Phi_t$$

$$\det D\Phi_t(x) = \exp\left(\int_0^t \nabla \cdot u(\Phi_s(x)) \, \mathrm{d}s \right) > 0$$

Exact flow: orientation preserved. Sampled numerical determinants: diagnostic only.

Rotation is an analytic benchmark. The lab fits a smooth initial velocity; it does not solve geodesic registration.