Mathematical Cardiology Lecture 3

Muscle force and the pressure–volume cycle

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Two levels of a pumping system

Crossbridges contribute force to muscle.
The ventricular wall turns tissue forces into chamber pressure.
A pressure–volume loop records chamber work, not individual crossbridge motion.

Add forces across a population

$$F = N \sum_j a_j f_j, \qquad a_j \geq 0, \quad \sum_j a_j \leq 1.$$

The attached fraction and each attached bridge's state both affect total force.

A load–speed family

$$v = b\frac{F_0 - F}{F + a}, \qquad a, b > 0.$$

At $F = 0$, $v_{\max} = bF_0/a$.
At $F = F_0$, $v = 0$.

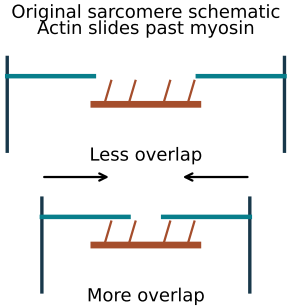
Make the variables dimensionless

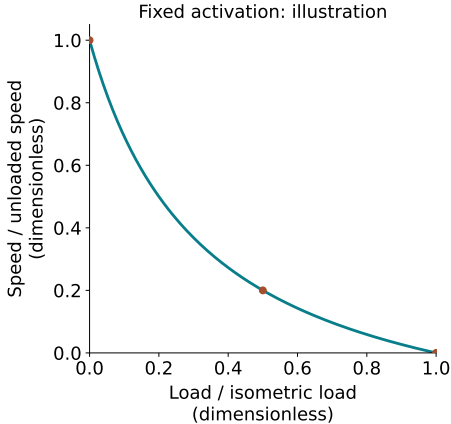
$$f = F/F_0, \quad u = v/v_{\max}, \quad \alpha = a/F_0.$$

$$u = \frac{\alpha(1-f)}{f+\alpha}.$$

For $\alpha = 1/3$, $u = (1-f)/(1+3f)$.

Force, speed, and power





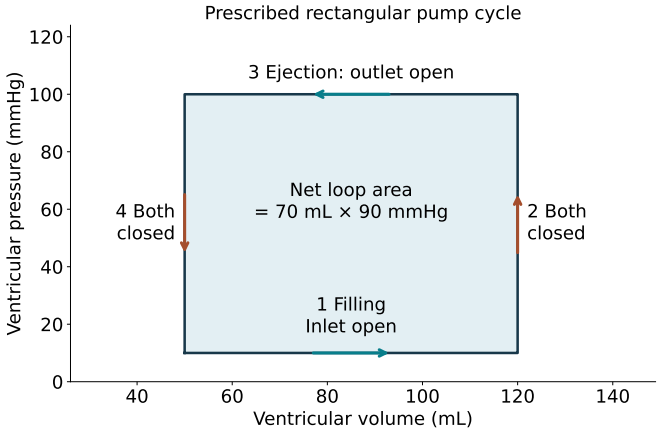
Force, speed, and power

$$f = \tfrac{1}{2} \Rightarrow u = \frac{1/2}{5/2} = \tfrac{1}{5}.$$

$$\text{normalized external power} = fu = \tfrac{1}{10}.$$

Zero shortening does not imply zero force or zero biochemical cost.

The PV plane


Prescribed rectangular pump cycle
3 Ejection: outlet open
Net loop area
= 70 mL × 90 mmHg
4 Both
closed
2 Both
closed
1 Filling
Inlet open
Ventricular pressure (mmHg)
Ventricular volume (mL)
0
20
40
60
80
100
120
40
60
80
100
120
140

The PV plane

Each point is (V, P) at one instant.
A directed path shows the order of states.
Distance along the path does not specify the duration of a phase.

Filling

$$V : 50 \longrightarrow 120 \text{ mL}, \qquad P = 10 \text{ mmHg}.$$

Mitral valve open; aortic valve closed in the idealized cycle.

Isovolumic contraction

$$V = 120 \text{ mL}, \qquad P : 10 \longrightarrow 100 \text{ mmHg}.$$

Both valves closed.

Fixed chamber volume does not mean inactive muscle.

Ejection and relaxation

At high pressure, volume falls from 120 to 50 mL.
Then both valves close and pressure falls at fixed volume.
The cycle returns to its starting state.

The ejected volume

$$SV = EDV - ESV = 120 - 50 = 70 \text{ mL/beat}.$$

The retained 50 mL is part of the next cycle's starting inventory.

A fraction is not an efficiency

$$EF = \frac{SV}{EDV} = 1 - \frac{ESV}{EDV} = \frac{70}{120}.$$

Scale both chamber volumes by two: EF stays fixed, while SV doubles.

Per beat to per minute

$$CO = (70\ \text{mL/beat})(75\ \text{beats/min})$$

$$= 5250\ \text{mL/min} = 5.25\ \text{L/min}.$$

The calculation assumes identical repeated beats.

Why pressure times volume is work

$$\text{work} = (\text{force})(\text{distance}) = (PA)\Delta x.$$

$$A\Delta x = \Delta V_{\text{displaced}}.$$

Hence work is $P\Delta V_{\text{displaced}}$.

The sign of chamber work

During ejection, chamber $\Delta V < 0$.

$$W_{\text{delivered}} \approx -P\Delta V.$$

Ejection contributes positively; filling contributes negatively under this convention.

Work around the rectangle

$$W = 100(70) - 10(70)$$

$$= (100 - 10)(120 - 50) = 6300 \text{ mmHg mL}.$$

The vertical segments contribute zero chamber-volume work.

Convert the area to energy

$$6300(1.33322 \times 10^{-4}) \approx 0.840 \text{ J}.$$

For a curved loop, add $-P_j \Delta V_j$ over small segments.
This mechanical output is not total metabolic expenditure.

Coupling force, geometry, and pressure

A force–velocity curve alone does not determine the PV loop.
The missing relations include activation, wall geometry, material response and vascular loading.
Measured pressure and volume must also be synchronized.