Mathematical Cardiology Lecture 3

Muscle force and the pressure–volume cycle

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Two levels of a pumping system

Crossbridges contribute force to muscle.

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The ventricular wall turns tissue forces into chamber pressure.

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The ventricular wall turns tissue forces into chamber pressure.
A pressure–volume loop records chamber work, not individual crossbridge motion.

Add forces across a population

$$F = N \sum_j a_j f_j, \qquad a_j \geq 0, \quad \sum_j a_j \leq 1.$$

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The attached fraction and each attached bridge's state both affect total force.

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At $F = F_0$, $v = 0$.

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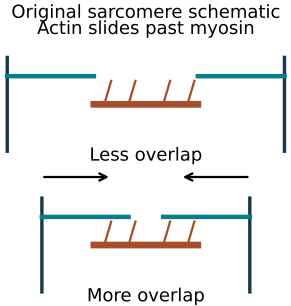
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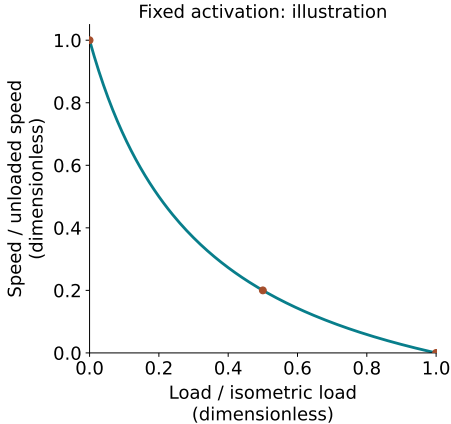
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$$u = \frac{\alpha(1-f)}{f+\alpha}.$$

For $\alpha = 1/3$, $u = (1-f)/(1+3f)$.

Force, speed, and power





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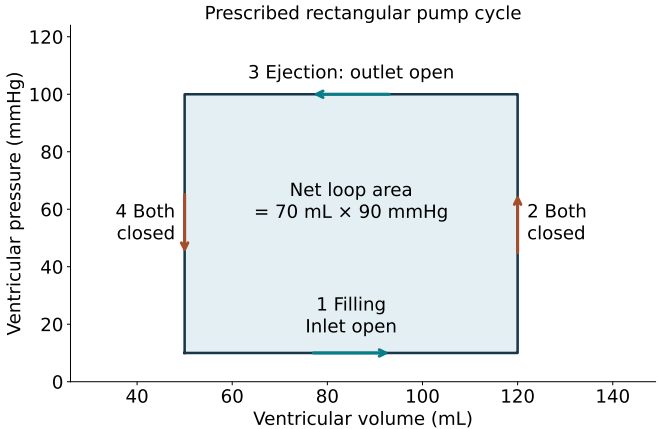
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$$\text{normalized external power} = fu = \tfrac{1}{10}.$$

Zero shortening does not imply zero force or zero biochemical cost.

The PV plane


Prescribed rectangular pump cycle
3 Ejection: outlet open
Net loop area
= 70 mL × 90 mmHg
4 Both
closed
2 Both
closed
1 Filling
Inlet open
Ventricular pressure (mmHg)
Ventricular volume (mL)
0
20
40
60
80
100
120
40
60
80
100
120
140

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A directed path shows the order of states.
Distance along the path does not specify the duration of a phase.

Filling

$$V : 50 \longrightarrow 120 \text{ mL}, \qquad P = 10 \text{ mmHg}.$$

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Mitral valve open; aortic valve closed in the idealized cycle.

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Fixed chamber volume does not mean inactive muscle.

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The cycle returns to its starting state.

The ejected volume

$$SV = EDV - ESV = 120 - 50 = 70 \text{ mL/beat}.$$

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The retained 50 mL is part of the next cycle's starting inventory.

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Scale both chamber volumes by two: EF stays fixed, while SV doubles.

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$$CO = (70 \text{ mL/beat})(75 \text{ beats/min})$$

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The calculation assumes identical repeated beats.

Why pressure times volume is work

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Hence work is $P\Delta V_{\text{displaced}}$.

The sign of chamber work

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Ejection contributes positively; filling contributes negatively under this convention.

Work around the rectangle

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The vertical segments contribute zero chamber-volume work.

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This mechanical output is not total metabolic expenditure.

Coupling force, geometry, and pressure

A force–velocity curve alone does not determine the PV loop.

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The missing relations include activation, wall geometry, material response and vascular loading.

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The missing relations include activation, wall geometry, material response and vascular loading.
Measured pressure and volume must also be synchronized.