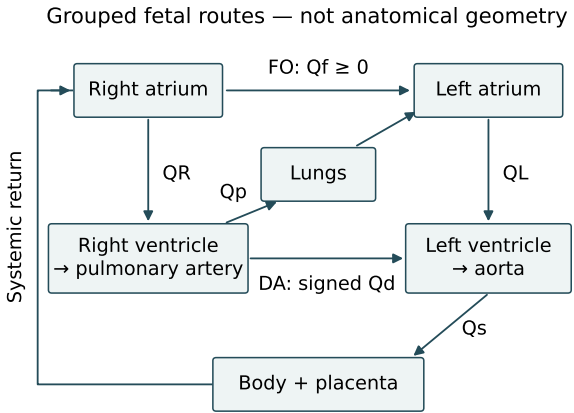
Mathematical Cardiology Lecture 7

Fetal pathways and signed junction balances

Zan Ahmad

Fall 2023

Two shunts change the routes



Two shunts change the routes

Retain pulmonary flow Q_p and grouped systemic flow Q_s. The foramen ovale joins the atria; the ductus arteriosus joins pulmonary artery and aorta.
The placenta is an exchange interface, not free mixing of maternal and fetal blood.

Declare the positive directions

$$Q_d > 0: \quad \text{pulmonary artery} \longrightarrow \text{aorta}.$$

$$Q_f > 0: \quad \text{right atrium} \longrightarrow \text{left atrium}.$$

A sign convention is not yet a one-way flow law.

At the pulmonary artery

Right-pump flow separates into two routes:

$$\boxed{Q_R = Q_p + Q_d}.$$

If $Q_p = 100$ and $Q_d = 50$, then $Q_R = 150\,\mathrm{mL/min}$.

At the left atrium and aorta

Pulmonary return and foramen flow enter the left side:

$$Q_L = Q_p + Q_f.$$

At the aortic junction,

$$Q_s = Q_L + Q_d.$$

The fourth balance is not independent

$$Q_s = Q_L + Q_d = (Q_p + Q_f) + Q_d.$$

Rearrange the sum:

$$Q_s = (Q_p + Q_d) + Q_f = Q_R + Q_f.$$

The right-atrial balance follows from the other three.

Pump output need not equal pulmonary flow

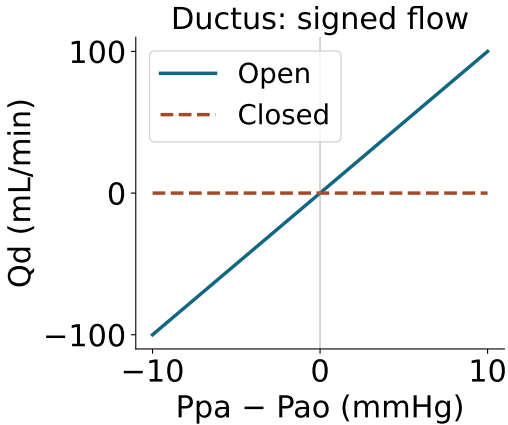
For $(Q_p, Q_d, Q_f) = (100, 50, 40)$,

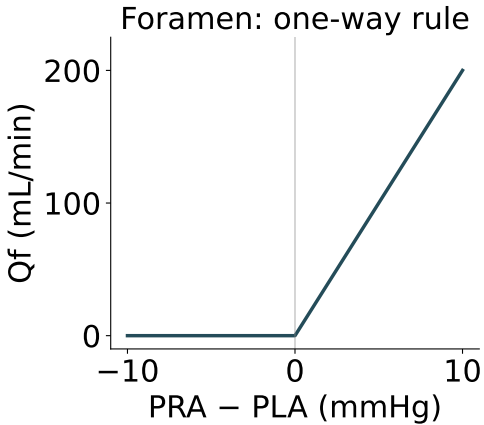
$$Q_R = 150, \qquad Q_L = 140, \qquad Q_s = 190\,\mathrm{mL/min}.$$

Check both systemic balances:

$$140 + 50 = 190 = 150 + 40.$$

An open ductus can reverse





An open ductus can reverse

$$Q_d = (P_{pa} - P_{ao})/R_d, \qquad R_d > 0.$$

For $R_d = 0.1$,

$$\frac{30-25}{0.1} = 50, \qquad \frac{20-30}{0.1} = -100\,\mathrm{mL/min}.$$

The second case carries blood from aorta to pulmonary artery.

A one-way rule is a different equation

$$Q_f = \frac{\max(P_{RA} - P_{LA}, 0)}{R_f}.$$

For $R_f = 0.05$,

$$Q_f(6,4) = 40, \qquad Q_f(4,7) = 0\,\mathrm{mL/min}.$$

Forward resistance is finite in this surrogate.

Zero flow does not establish closure

For a ductus,

$$Q_d = G_d(P_{pa} - P_{ao}).$$

$Q_d = 0$ may mean $G_d = 0$, or may mean equal boundary pressures.
Closure is zero conductance, not zero resistance.

Reversed flow enters the pulmonary junction

Take $Q_p = 300$, $Q_d = -100$, $Q_f = 0$.

$$Q_R = 300 - 100 = 200.$$

The missing $100\,\mathrm{mL/min}$ arrives from the aorta through the ductus.

The aortic balance agrees

$$Q_L = 300 + 0 = 300, \qquad Q_s = 300 - 100 = 200.$$

Of the left-pump output, 100 returns to the pulmonary route and 200 enters the systemic route.

No blood is created by the negative sign.

Conservation leaves freedom

Choose Q_p, Q_d, Q_f; then

$$\begin{pmatrix} Q_R \\ Q_L \\ Q_s \end{pmatrix} = \begin{pmatrix} Q_p + Q_d \\ Q_p + Q_f \\ Q_p + Q_f + Q_d \end{pmatrix}.$$

Conservation determines the remaining flows, but does not choose the first three.

Pressure rules do not solve the whole circulation

The companion prescribes local pressures and illustrative pulmonary flows. A full model must also determine those pressures from chambers, vascular properties, boundary conditions and blood volume.

The transition changes both pressures and routes

Pulmonary conditions change; the placental route is removed. Ductus flow can reverse while conductance remains positive. Foramen flow can stop under an unfavorable pressure difference.
Neither event alone specifies anatomical closure time.

Recover the series circulation as a limit

If $Q_d = Q_f = 0$,

$$Q_R = Q_p, \qquad Q_L = Q_p, \qquad Q_s = Q_L.$$

$$\boxed{Q_R = Q_p = Q_L = Q_s}.$$

Deleting a shunt requires rebalancing the flows.