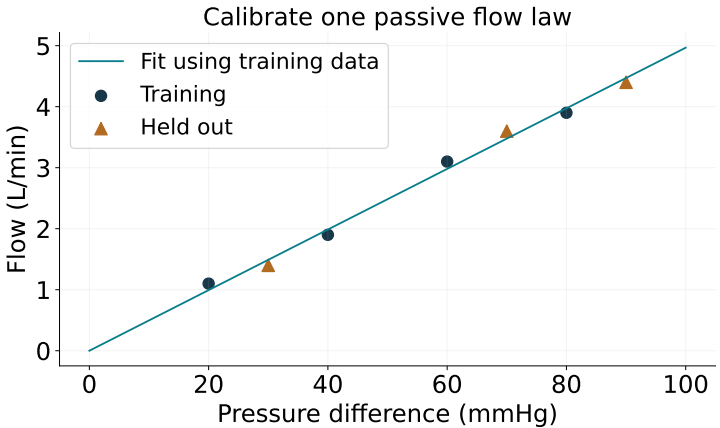
Mathematical Cardiology Lecture 8

Calibrating a flow law and constructing a classifier

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A parameter inside a physical relation



A parameter inside a physical relation

$$\widehat{Q} = G\Delta P, \qquad R = 1/G.$$

G has units $\mathrm{L}/(\min \mathrm{mmHg})$. A line through the origin asserts zero passive flow at equal boundary pressures.

Define the discrepancy

For training pairs (x_i, y_i),

$$r_i(G) = Gx_i - y_i, \qquad S(G) = \sum_i r_i(G)^2.$$

Opposite signed errors do not cancel after squaring. The objective has units $(\mathrm{L/min})^2$.

Expand the objective

$$S(G) = \sum_i (G^2 x_i^2 - 2G x_i y_i + y_i^2).$$

Write

$$S(G) = AG^2 - 2BG + C,$$

where $A = \sum x_i^2$, $B = \sum x_i y_i$, $C = \sum y_i^2$.

Complete one square

When $A > 0$,

$$S(G) = A(G - B/A)^2 + C - B^2/A.$$

The square is smallest at zero, so

$$\boxed{\widehat{G} = B/A}.$$

Evaluate the fitted conductance

For the four displayed training observations,

$$A = 12000, \qquad B = 596.$$

$$\widehat{G} = 0.049667, \qquad \widehat{R} = 20.134\,\mathrm{mmHg\,min/L}.$$

These digits describe this synthetic sample.

An experiment with no slope information

If every training pressure difference is zero,

$$A = 0, \qquad S(G) = \sum_i y_i^2.$$

Every G gives the same objective. More zero-pressure observations do not identify conductance.

A unit conversion check

Let $x_i' = cx_i$ with $c > 0$.

$$\widehat{G}' = \frac{c \sum x_i y_i}{c^2 \sum x_i^2} = \widehat{G}/c.$$

$$\widehat{G}' x_i' = \widehat{G} x_i.$$

The prediction stays the same.

Evaluate the frozen line

x	30	70	90
y	1.4	3.6	4.4
$\widehat{y}$	1.4900	3.4767	4.4700
r	0.0900	-0.1233	0.0700

$\mathrm{MAE} \simeq 0.09444\,\mathrm{L/min}.$

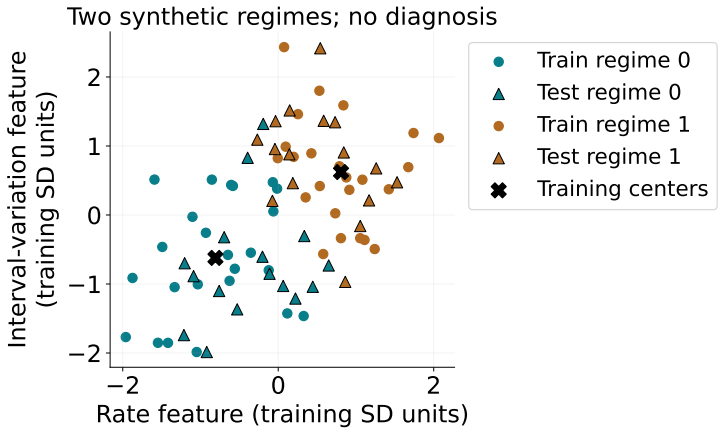
Compare with a constant predictor

The training mean is $2.5\,\mathrm{L/min}$. Its held-out absolute errors are 1.1, 1.1, 1.9.

$$\mathrm{MAE}_{\mathrm{constant}} = 4.1/3 \simeq 1.3667\,\mathrm{L/min}.$$

Both rules are evaluated on the same three cases.

Fit a coordinate system on training cases



Fit a coordinate system on training cases

For each feature,

$$z_j = (x_j - \mu_j)/s_j.$$

Compute μ_j, s_j from the 48 training cases. Use those same values for the 32 held-out cases.

The nearer training center

Let c_0, c_1 be the class means in standardized coordinates.

$$d_k(z) = \sum_{j=1}^{2} (z_j - c_{kj})^2.$$

Choose the smaller d_k; exact ties go to zero in this implementation.

Expand the decision boundary

Label one requires

$$\|z - c_1\|^2 < \|z - c_0\|^2.$$

The $\|z\|^2$ terms cancel:

$$2z \cdot (c_1 - c_0) > \|c_1\|^2 - \|c_0\|^2.$$

Equality is a line when the centers differ.

Scaling determines the metric

In raw coordinates,

$$d_k(x) = \sum_{j=1}^{2} \frac{(x_j - \overline{x}_{kj})^2}{s_j^2}.$$

The training variations determine the weights. A new test mean would change the fitted rule.

Read both directions of error

true\predicted	0	1
0	12	4
1	0	16

Accuracy is $28/32 = 0.875$. All four errors are $0 \to 1$.

The held-out unit belongs to the claim

A claim about new people requires whole people to be held out. Windows from one person can share information even if the windows differ.
Choose the grouping first; fit scaling and the rule on training groups.

After inspecting an error

Changing the model in response to test errors makes those cases development data.
Retain the original result. Evaluate the changed rule on other observations.
A transparent fit is useful because its assumptions and failures can be named.