Scientific Computing in Python / 2027 Lecture 1

Coordinates, sampled motion, and Python expressions

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Proposed 2027; calendar dates pending

Position and displacement

$$t = (0, 2, 4, 6, 8), \quad x_A = (10, 14, 18, 22, 26), \quad x_B = (0, 6, 12, 18, 24).$$

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At $t = 8$, $x_A > x_B$, but $\Delta x_A = 16 < 24 = \Delta x_B$.

Average velocities

$$\overline{v}_A = \frac{26 - 10}{8 - 0} = 2 \text{ mm/s}, \qquad \overline{v}_B = \frac{24 - 0}{8 - 0} = 3 \text{ mm/s}.$$

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Final position and average velocity answer different comparisons.

Assignment and evaluation

```
a_mm = [10, 14, 18, 22, 26]
displacement = a_mm[-1] - a_mm[0]
elapsed = 8 - 0
velocity = displacement / elapsed
```

Reassignment

```
a = 10
b = a + 2
a = 20
```

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b = a + 2
a = 20
```

The final values are $a = 20$ and $b = 12$. Python has executed statements, not solved simultaneous equations.

Translation of the coordinate origin

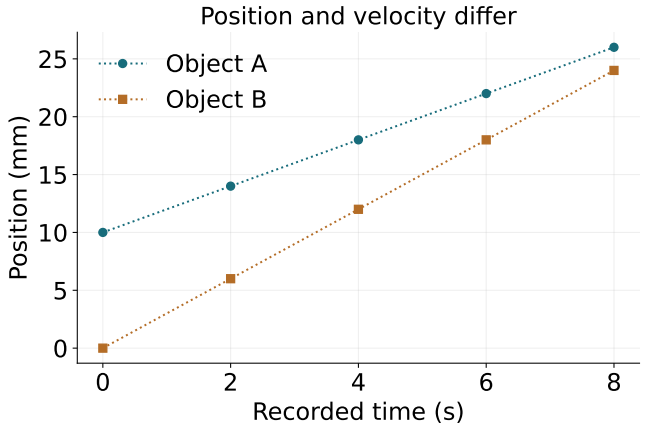
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$$\frac{\Delta x'}{(t_1 + d) - (t_0 + d)} = \frac{\Delta x}{\Delta t}.$$

Position records



An invariance test

```
shifted = (26 + 100) - (10 + 100)
assert shifted == 26 - 10
```

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assert shifted == 26 - 10
```

A failed test indicates that the code depends on the origin in a way that the defined statistic does not.

Change of physical units

$$x' = ax, \quad t' = bt \quad \Longrightarrow \quad \overline{v}' = \frac{a}{b}\overline{v}.$$

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$$2 \text{ mm/s} = 0.2 \text{ cm/s} = 12 \text{ cm/min}.$$

Numerical comparison after conversion

```
dx_mm = 26 - 10
dx_cm = 26/10 - 10/10
assert abs(10*dx_cm - dx_mm) < 1e-12
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```

The tolerance accounts for rounding, not measurement uncertainty.

Displacement and path length

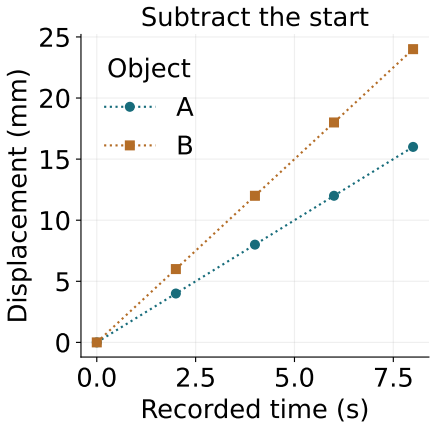
$$\Delta x = x_N - x_0 = \sum_{i=0}^{N-1} (x_{i+1} - x_i).$$

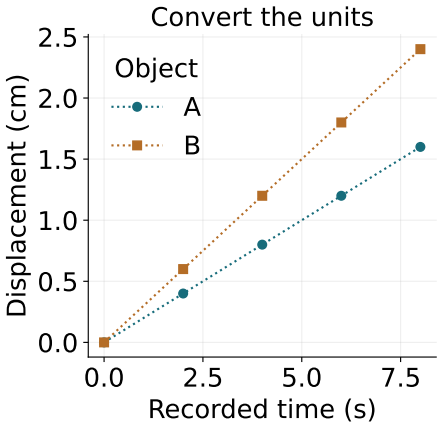
Displacement and path length

$$\Delta x = x_N - x_0 = \sum_{i=0}^{N-1} (x_{i+1} - x_i).$$

$$|\Delta x| \le \sum_{i=0}^{N-1} |x_{i+1} - x_i| = L_{\text{sampled}}.$$

Displacement units





An out-and-back record

For positions $0, 4, 0$ at times $0, 1, 2$,

$$\Delta x = 0, \qquad L_{\text{sampled}} = 8.$$

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The signed average velocity is zero. The sampled mean speed is 4 distance units per time unit.

Information between samples

Two trajectories can agree at every sample time while differing between them.

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Two trajectories can agree at every sample time while differing between them. For a smooth interpolant $x(t)$, add

$$g(t) = c \prod_i (t - t_i).$$

The sampled positions stay fixed.

Array order and data correspondence

```
assert len(times_s) == len(a_mm)
plt.plot(times_s, a_mm, "o")
```

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```

The ith time and ith position must belong to the same observation. Equal lengths alone do not prove correct pairing.