Scientific Computing in Python / 2027 Lecture 2

Functions, array axes, and planar trajectories

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Proposed 2027; calendar dates pending

The axes of a trajectory array

$$p \in \mathbb{R}^{R \times T \times 2}, \qquad p_{ri0} = x_{ri}, \quad p_{ri1} = y_{ri}.$$

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Time is axis 1. Component is axis 2.

Indexing an observation

```
p[r, i, :]       # one two-component position
p[r, :, :]       # one full trajectory
p[:, i, :]       # all positions at one time
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Each slice retains a different physical question.

Interval-average velocity

$$v_{ric} = \frac{p_{r,i+1,c} - p_{ric}}{t_{i+1} - t_i}.$$

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The result has shape $(R, T-1, 2)$.

Broadcasting the time increments

```
dp = np.diff(p, axis=1)
dt = np.diff(t)
v = dp / dt[None, :, None]
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The two singleton axes repeat the same interval durations across trajectories and coordinate components.

Magnitude and threshold indicators

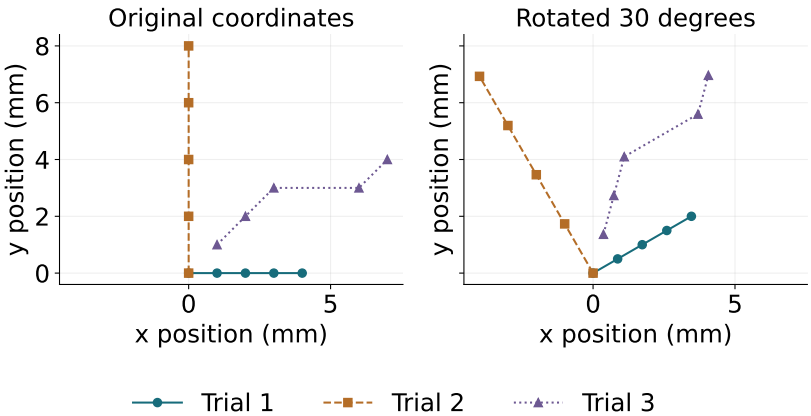
$$s_{ri} = \sqrt{v_{ri0}^2 + v_{ri1}^2}, \qquad I_{ri} = \mathbf{1}_{\{s_{ri} > s_*\}}.$$

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The strict inequality excludes equality.

Trajectory rotation



A threshold function

```
def above_threshold(speed, target):
    return speed > target
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At $s_* = 2$, a speed of 2 does not qualify and 2.01 does. Missing and nonfinite inputs require a separate policy.

A rotation matrix

$$R(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}.$$

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$p' = Rp$ for columns, $P' = PR^{\mathsf{T}}$ for rows.

Rotation of a basis vector

At $\theta = \pi/6$,

$$R\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}\sqrt{3}/2\\1/2\end{pmatrix}.$$

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$$R \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \sqrt{3}/2 \\ 1/2 \end{pmatrix}.$$

Its length remains one, while its coordinate components change.

Orthogonality and length preservation

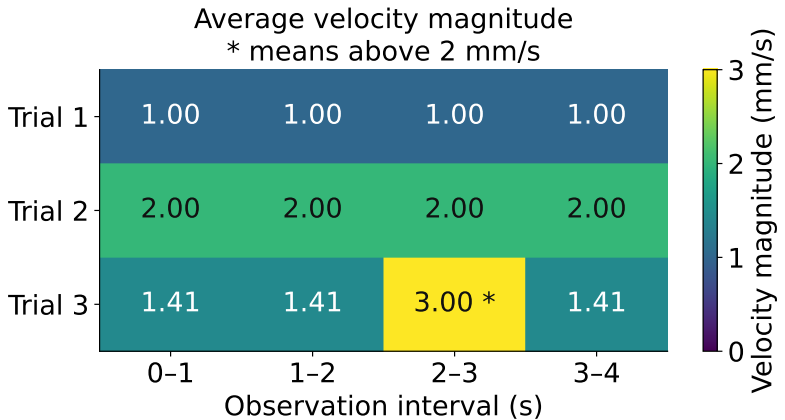
$$\|R\Delta p\|_2^2 = \Delta p^{\mathsf{T}} R^{\mathsf{T}} R \Delta p = \|\Delta p\|_2^2.$$

Orthogonality and length preservation

$$\|R\Delta p\|_2^2 = \Delta p^{\mathsf{T}} R^{\mathsf{T}} R \Delta p = \|\Delta p\|_2^2.$$

Thus every interval speed is unchanged.

Speed shape map



Commutation with finite differences

$$\frac{Rp_{i+1} - Rp_i}{\Delta t_i} = R\frac{p_{i+1} - p_i}{\Delta t_i} = Rv_i.$$

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A discrepancy beyond floating-point tolerance suggests an axis or transpose error.

Functions with explicit input conditions

Input conditions include

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Check both the input values and the computed velocities for finiteness.

A geometric test suite

Translation leaves velocity unchanged. Rotation leaves speed unchanged. Rescaling length by a rescales speed by $|a|$.

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Translation leaves velocity unchanged. Rotation leaves speed unchanged. Rescaling length by a rescales speed by $|a|$. These relations test the mathematical operation over many arrays without hard-coding every result.