

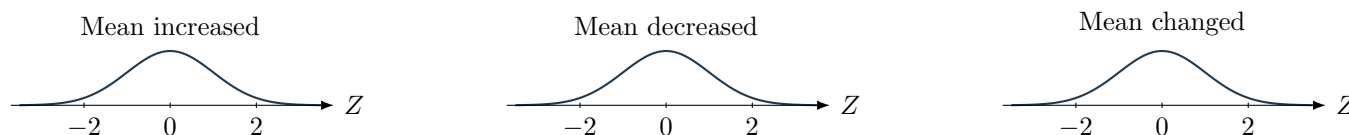
Chapter 8 Problem 1

The GPA Example Student notes, pp. 5–6.

Use the example from the recorded Chapter 8 lecture. The historical reference mean GPA is 3.37. An independent random sample of 50 current students has $\bar{x} = 3.47$ and $s = .31$. Before collecting the sample, the research question was whether the population mean GPA had increased. The chosen significance level is $\alpha = .05$.

Use the lecture's large-sample normal approximation, assuming its sampling conditions are adequate. The population standard deviation is unknown. Write Φ for the standard normal CDF.

- Hypotheses and parameters.** Define the population parameter μ . Explain the roles of the *null hypothesis* and *alternative hypothesis*, and state both for this question. Identify the *null value* μ_0 . Is a hypothesis a statement about μ or about the observed $\bar{x}$?
- Statistic and reference distribution.** Write the test statistic in terms of the random quantities $\bar{X}$ and S , then calculate its observed value z using $\bar{x}$ and s . Explain what the numerator and denominator measure. Define the *null distribution*; state the distribution used here and the assumption under which it applies. Which is random before sampling: the statistic or the null value?
- The p -value and its tail.** Define a p -value in words, including both what is assumed and what counts as “at least as favorable.” For each separately specified research question below, state H_a , write a probability involving Z and the observed z , shade its region, and calculate the p -value. The data stay the same; these are three alternative questions, each chosen *before* seeing data.



Mark the observed boundary or boundaries and shade the relevant region. Use a normal table or calculator.

- Significance and conclusion.** Define the significance level as used so far in this lecture. For the original *increase* question, state the decision at .05 and the conclusion about the population. Reassess the same evidence using a prechosen .01 level. Which of the data, z , p -value, threshold, and decision change?
- Interpretation check.** Explain what is wrong with each statement and replace it with a correct one:
 - “The p -value is the probability that our null hypothesis is wrong.”
 - “If we fail to reject, we have proved that the population mean is 3.37.”

Worked Solutions

(a) A claim about a population parameter

μ is the mean GPA of the current student population from which the random sample is drawn. The null hypothesis supplies the reference claim $H_0 : \mu = 3.37$. The research question supplies $H_a : \mu > 3.37$. The null value is the fixed number $\mu_0 = 3.37$. Hypotheses concern μ ; $\bar{x} = 3.47$ is an observed estimate, not the parameter.

(b) Random statistic and observed value

$$Z = \frac{\bar{X} - 3.37}{S/\sqrt{50}} \stackrel{\text{approx}}{\sim} N(0, 1) \text{ under } H_0, \quad z = \frac{3.47 - 3.37}{.31/\sqrt{50}} \approx 2.28099.$$

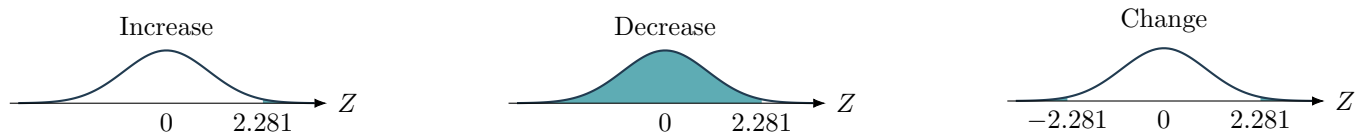
The numerator is the sample mean's discrepancy from the null value. The estimated sampling SE is $.31/\sqrt{50} \approx .0438406$ GPA units. The observed mean is about 2.281 SEs above the null value. Before sampling, $\bar{X}$, S , and Z vary; the stipulated null value does not. After sampling, $\bar{x}$, s , and z are fixed observations.

The *null distribution* is the reference distribution of the statistic when H_0 and the sampling assumptions hold. Here it is approximately standard normal under the lecture's large-sample method. Substituting an estimated SD does not make this an exact standard-normal result. The sample-size rule is a course convention, not a universal guarantee for arbitrary populations.

(c) Evidence depends on the alternative

A p -value is the probability, assuming H_0 and the model, of a statistic at least as favorable to the *specified* H_a as the observed statistic. With $z = 2.28099$:

Alternative	Tail probability	Approximate p -value
$\mu > 3.37$	$P(Z \geq z) = 1 - \Phi(z)$	.01127
$\mu < 3.37$	$P(Z \leq z) = \Phi(z)$	.98873
$\mu \neq 3.37$	$P(Z \geq z) = 2[1 - \Phi(z)]$	.02255



(d) Thresholds and conclusions

So far, α is the *prechosen threshold* for the p -value. Reject when $p\text{-value} \leq \alpha$; otherwise fail to reject. For the increase question, $.01127 < .05$: there is evidence of an increased population mean GPA. At .01, fail to reject: the evidence does not meet that stricter threshold. The data, observed z , and p -value stay the same. Only the threshold and decision change.

(e) Repair the interpretations

(i) Under the null model, about 1.13% of samples would yield a standardized increase at least this large. The p -value is not the probability that H_0 is wrong (or true). The null model is assumed when this tail probability is calculated.

(ii) Failure to reject means insufficient evidence for the specified alternative at that level; it does not prove equality.