Comparing germination proportions

Chapter 8 practice Hypothesis testing and confidence intervals

Two independent random samples each contain 100 seeds grown under one of two methods. Under method A, 60 germinate; under B, 45 germinate. Is there evidence that the population germination proportions differ? Use $\alpha = .05$.

	Method A	Method B
Seeds sampled	100	100
Seeds that germinate	60	45

Assume independent Bernoulli outcomes within and between the samples, with a constant germination probability within each method. Let p_A and p_B denote the population germination probabilities.

Useful information. The upper-tail normal critical value is $z_{.025} = 1.960$. You can make the testing decision using this cutoff. If you also calculate a p-value, use a standard normal table or calculator.

(a) State the null and alternative hypotheses. Calculate the two sample proportions and the estimated difference $p_A - p_B$.

(b) Carry out the appropriate two-proportion test at the 5% significance level. Explain why pooling is appropriate under this null, check the null expected-count conditions, calculate the standard error and test statistic, and state your conclusion in context. Include an approximate p-value if a normal table or calculator is available.

(c) Construct an approximate 95% confidence interval for $p_A - p_B$. Explain why the interval uses separate sample proportions in its standard error. Does its conclusion agree with the test in this example? Does that agreement make the pooled test and this interval exact inverses in general?

(d) Describe a Type I error and a Type II error in the context of this comparison. Explain what the power of this test would mean at a specified pair of unequal population proportions. Is a p-value the probability that the null hypothesis is true?

Show your reasoning. Identify the population quantities, the sampling uncertainty, and which discrepancies count as evidence. Report the estimated difference in percentage points as well as a testing decision.

Worked solution

(a) Parameter and hypotheses.

$$H_0 : p_A = p_B, \qquad H_a : p_A \neq p_B, \qquad \hat{p}_A = .60, \quad \hat{p}_B = .45, \quad \hat{p}_A - \hat{p}_B = .15.$$

The estimated difference is 15 percentage points, with the higher observed germination rate under method A.

(b) Test under the equality null. Under H_0 both groups share one unknown success probability. Its pooled estimate is

$$\hat{p} = \frac{60 + 45}{100 + 100} = .525.$$

The fitted null expected counts in each group are $100(.525) = 52.5$ successes and $100(.475) = 47.5$ failures. These exceed the course guideline of 5. Along with the stated independence assumptions, this supports a normal approximation.

$$\widehat{\mathrm{SE}}_0 = \sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{100} + \frac{1}{100}\right)} = \sqrt{.525(.475)(.02)} = .0706222,$$

$$z = \frac{(.60 - .45) - 0}{.0706222} = 2.1240.$$

Since $|z| > 1.960$, reject H_0 at $\alpha = .05$. Equivalently, with Φ the standard normal CDF,

$$p\text{-value} \approx 2[1 - \Phi(2.1240)] = .03367 < .05.$$

There is evidence that the population germination proportions differ, under the sampling model. The estimate favors method A; the result does not itself establish a causal effect.

(c) Confidence interval without imposing equality. For estimation, retain separate probabilities and estimate their variances separately:

$$\widehat{\mathrm{SE}}_{\mathrm{CI}} = \sqrt{\frac{.60(.40)}{100} + \frac{.45(.55)}{100}} = .0698212,$$

$$95\%\text{ Wald CI}: \quad .15 \pm 1.960(.0698212) = \boxed{(.01315,\ .28685)}.$$

The approximate interval ranges from 1.32 to 28.68 percentage points and excludes zero, agreeing with the test here. The pooled null test and unpooled Wald interval are not exact inverses: they use different standard errors. Matching CI/test equivalence requires compatible methods and levels.

(d) Error probabilities and power. A Type I error would conclude that the proportions differ when they are actually equal. A Type II error would fail to detect a difference when the true proportions are unequal. At specified true values (p_A, p_B), power is the probability that this same test rejects equality:

$$\mathrm{Power}(p_A, p_B) = 1 - \Pr(\text{fail to reject } H_0 \mid p_A, p_B).$$

The value depends on the specified alternative and the sampling model. The p-value measures how unusual the observed discrepancy or a more extreme one would be under the null model; it is not the probability that H_0 is true. The normal test's Type I calibration is approximate.